Erosion of Sand by Circular Impinging Water Jets with Small Tailwater

N. Rajaratnam, F.ASCE,1 and K. A. Mazurek, M.ASCE2

Abstract: This technical note presents the results of an experimental study of the erosion of loose cohesionless sand beds by impinging circular water jets with a minimum depth of tailwater. Measurements were made of both the maximum dynamic and static scour depths and the radius of the scour hole. It was found that the dynamic scour depth is about three times that of static scour at the asymptotic state. Dimensional arguments and experimental results are used to show that the main dimensions of the scour hole at the asymptotic state are a function of the densimetric Froude number \( F_0' = U_0' \sqrt{gDp/\rho} \), where \( U_0' \) = velocity of the jet at the original level of the sand bed; \( g \) = acceleration due to gravity; \( D \) = mean diameter of the sand particles; \( \rho \) = density of the eroding fluid; and \( \Delta p \) = difference between particle and fluid densities. Useful correlations have been developed to estimate the size of the scour holes. Also included is a comparison between the erosion caused by submerged and unsubmerged impinging circular jets.

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Introduction

Erosion of sand, gravels, and other materials, which often occurs downstream of hydraulic structures, is of considerable importance, as excessive scour may endanger the stability of these structures, and it has frequently been modeled as erosion by jets. Many studies have been performed to examine the erosion of cohesionless materials by submerged circular turbulent impinging jets, where the tailwater depth is large, including those of Doddiah et al. (1953), Westrich and Kobus (1973), Rajaratnam and Beltaos (1977), Mih and Kabir (1983), and Aderibigbe and Rajaratnam (1996). It is known that the erosion characteristics of a scour hole produced by a submerged jet set at a large impingement height \( H \) (the height of the jet above the soil bed) is different from the scour produced when the jet is close to the bed. For large impingement heights, the maximum depth of scour \( e_{max} \) is properly scaled with \( H \), and \( e_{max}/H \) is mainly a function of \( F_0'/(H/d) \), where \( F_0' = U_0' \sqrt{gD\Delta p/\rho} \), and \( U_0 \) and \( d \) are the velocity and diameter of the jet at the nozzle. However, for small impingement heights, the maximum depth of scour scales with \( d \) as \( e_{max}/d \) and is a function of \( F_0' \) (Rajaratnam and Beltaos 1977). It is considered that the tailwater depth is yet another important variable for scour by these jet flows. This experimental study constitutes an extension of the previous work on submerged jets and considers the case of erosion of cohesionless sand beds by circular water jets when the tailwater depth is very small.

Experiments and Experimental Results

For the experiments, a large octagonal tank, of 572 mm width and 610 mm height, was filled with sand to the top of the tank. This sand was always kept fully saturated. An unsubmerged circular water jet was set to impinge at 90° to the sand bed. The jet was created by flow through a long plenum of 120 mm inside diameter that issued through either of two well-designed nozzles of 9.8 or 12.7 mm diameter. The nozzle was set at a height \( H \) above the sand bed and was centrally located above the octagonal tank. Fig. 1 shows the experimental setup. It is similar to that used by Mazurek et al. (2001) in experiments on scour by vertically impinging jets in a cohesive material. A total of 18 experiments were performed using three types of sand with a mean size \( D \) (50% of the material by weight was finer than this size) equal to 1.0, 1.15, and 2.38 mm. For all experiments, the depth of tailwater was very small and was maintained by the radially spreading flow itself. Details of the experiments are given in Table 1.

In the first few experiments, the static erosion profiles were measured using a point gauge at a number of time intervals until the asymptotic state was reached. The asymptotic state of scour occurs after long times as the scouring rate becomes very small (Doddiah et al. 1953; Westrich and Kobus 1973), when for all practical purposes the scour hole can be assumed to have reached its final or largest size. Fig. 2 shows the growth of the maximum static scour \( e_{max} \) with time for Experiment 13. It is seen that for a large portion of the growth of the scour hole \( e_{max} \) grew in a linear relation with the logarithm of time, but departed from this trend as the scour hole neared the asymptotic state (as is typically seen for submerged jets). As it was found in the first experiments that the time required for the asymptotic state to be established was

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about one day, the rest of the experiments were run for a duration of at least one day and only the static scour profiles in the asymptotic state were measured. Fig. 3 shows some typical asymptotic scour profiles. From these profiles, the maximum depth of scour $\varepsilon_{m}$ and the scour hole radius $r_{0}$ were measured as defined in Fig. 4. An interesting observation is the absence of a large ridge of sand around the edge of the scour hole, which occurs for scour holes formed in cohesionless materials for large tailwater depths (Rajaratnam and Beltaos 1977; Aderibigbe and Rajaratnam 1996). A flattened ridge or nonexistent ridge has also been observed in studies of the erosion by plane impinging jets under low tailwater conditions (Rajaratnam 1982; Akashi and Sai-tou 1986).

Observations of the characteristics of the jet interaction with the bed showed that for certain conditions the jet appears to entrain the sand in such a way as to set up an annular recirculating region in the sand surrounding the jet. Under other conditions, the impinging jet, after plunging into the sand bed, is reflected back upon itself to reappear above the original sand surface as a fountain loaded with sand (Fig. 5). Due to this behavior, the dynamic scour profiles (the scour profile when the jet is on) were distinctly different from the static scour profiles that appeared when the jet was stopped. Thus for the second and third series of experiments (21–27 and 31–37), an effort was made to determine the maximum depth of scour for dynamic scour as well as for the static scour case. The maximum dynamic scour depth $\varepsilon_{m}'$ was measured using a special lighting technique and by prodding with a rod within the scour hole to find where the bed was not fluidized. It should be mentioned, however, that for some experiments, particularly those with large jet velocities, $\varepsilon_{m}'$ could be measured only very approximately.

### Table 1. Details of Experiments

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<th>$U_0'$ (m/s)</th>
<th>$d'$ (mm)</th>
<th>$F_0'$</th>
<th>$R'$ ($10^4$)</th>
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With reference to Fig. 4, if $U'_0$ and $d'$ are, respectively, the mean velocity and diameter of the water jet at the level of the uneroded sand bed, using the Bernoulli and continuity equations, it can be shown that

$$U'_0 = \sqrt{U^2_0 + 2gH}$$  \(1\)

$$d' = d \frac{U'_0}{U'_0'}$$  \(2\)

For the maximum depth of static scour $e_{m=\infty}$, one can write

$$e_{m=\infty} = f_1\{U'_0', d', p, \Delta \rho, D, \nu\}$$  \(3\)

where $\nu$ = kinematic viscosity of the eroding fluid. Herein, the potential effects of nonuniformity in the gradation of the soil and the angle of repose of the material are neglected. Using the Pi theorem, Eq. (3) becomes

$$e_{m=\infty}/d' = f_3\{F'_0\}$$  \(4\)

For large values of the Reynolds number $U'_0d'/\nu$ (greater than a few thousand) and for large values of $d'/D$, Eq. (4) may be reduced to

$$e_{m=\infty}/d' = f_3\{F'_0\}$$  \(5\)

Following similar reasoning, it can be shown that

$$r_{0=\infty}/d' = f_4\{F'_0\}$$  \(6\)

The relation between the depth of erosion at the asymptotic state, in the dimensionless form $e_{m=\infty}/d'$, and $F'_0$ is shown in Fig. 6. Although there is appreciable scatter, a mean line is drawn which indicates that $e_{m=\infty}/d'$ increases linearly with $F'_0$ and is given by

$$e_{m=\infty}/d' = 0.13F'_0$$  \(7\)

The scatter in the data is perhaps due to slightly varying tailwater levels in the experiments. The ratio of the dynamic scour to the static scour in the asymptotic state $e'_m/e_{m=\infty}$ was also plotted against $F'_0$, as shown in Fig. 7. It is difficult to discern any influence of $F'_0$ on $e'_m/e_{m=\infty}$, and $e'_m/e_{m=\infty}$ is 3. For the scour hole radius, the results were plotted in Fig. 8, as $r_{0=\infty}/d'$ against $F'_0$. It is found that the scour hole radius can be estimated using the equation

$$r_{0=\infty}/d' = 0.72F'_0 - 6.40$$  \(8\)

with an $R^2$ of 0.84 for the range of $F'_0$ in these experiments.

Let us now compare the static erosion caused by an unsubmerged impinging jet, $e_{m=\infty}$, studied herein, with that produced by a submerged jet, $e_{m=\infty}$ (Fig. 9). The values used for $e_{m=\infty}$ are the experimental values reported here, whereas the values for $e_{m=\infty}$ are based on the equations presented by Aderibigbe and Rajaratnam (1996) for jets at large impingement heights. It is seen that $e_{m=\infty}/e_{m=\infty}$ varies linearly from about 0.3 for $F'_0/(H/d) = 0.4$ to 1.0 for $F'_0/(H/d) = 2.1$. Thus it appears that the static scour for
the unsubmerged case is less than the static scour for the submerged jet case for $0.4 < F_o / (H/d) < 2.1$. For larger values of $F_o / (H/d)$, the unsubmerged jet produces a deeper scour than the submerged jet. This behavior is likely due to the varying scour regimes for the submerged jet. At low values of $F_o / (H/d)$, the submerged jet is weakly deflected, and there is not much sediment that stays suspended within the scour hole while the jet is flowing to settle out in the scour hole when the jet flow is stopped (creating the static scour depth). At higher values of $F_o / (H/d)$, the jet becomes more and more strongly deflected, until the walls of the scour holes are almost vertical and the jet is turned back on itself. In the strongly deflected regime, there is a large amount of sediment that stays suspended within the recirculating flow within the scour hole, which settles out into the scour hole when the jet is stopped. Thus, the unsubmerged jet, where the jet is always strongly deflected, may create less static scour as compared to the submerged jet case, particularly at low $F_o / (H/d)$, due to the settling out of the sediment that stays suspended within the scour hole during jet flow (i.e., there is less material actually transported out of the scour hole for the unsubmerged jet case). For higher $F_o / (H/d)$, the submerged jet is also strongly deflected and this effect diminishes.

The ratio of the radius of the scour hole for an unsubmerged jet to the submerged jet, $r_{0x} / r_{0sx}$ (also seen in Fig. 9), increases from about 0.6 to 1.6 over a range of $F_o / (H/d)$ of 0.4 to 1.6. The

**Fig. 7.** Ratio of dynamic to static maximum scour depth at asymptotic state

**Fig. 9.** Comparison of scour due to unsubmerged and submerged impinging jets

unsubmerged jet produces a smaller scour hole than the submerged jet for $F_o / (H/d) < 1$, likely because the unsubmerged jet is more strongly deflected than the submerged jet, resulting in a smaller scour hole radius. Here the values of $r_{0s*}$ are also calculated from the equations presented by Aderibigbe and Rajaratnam (1996). As both the unsubmerged and the submerged jets become strongly deflected at higher values of $F_o / (H/d)$, the scour hole radius may be larger than for the submerged jet case because of the effect of the shallow tailwater, which results in a smaller or nonexistent ridge and thus a greater spreading of the sand that is transported out of the scour hole in the radial direction away from the jet.

**Conclusions**

Based on an experimental study of the erosion of cohesionless sand beds by unsubmerged impinging circular water jets, the following conclusions can be drawn. There are two distinct states of erosion, the dynamic scour that exists when the jet is on and the static scour that appears after the jet is turned off. Considering the static erosion, the maximum depth of erosion increases linearly with the logarithm of time for a large part of the erosion process and eventually reaches an asymptotic or end state. The asymptotic value of the erosion depth $\varepsilon_{m*}$ as well as the radius of the scour hole $r_{0*}$ in terms of the diameter of the impinging jet at the level of the original uneroded bed are mainly functions of the densimetric Froude number $F'_0$, where $F'_0$ is based on the velocity of the jet at the original sand level. The dynamic scour $\varepsilon'_{m*}$ was found to be about three times the static scour $\varepsilon_{m*}$ in the asymptotic state. It appears that the maximum depth of scour produced by an unsubmerged jet is less than that for a submerged jet for $F'_0 / (H/d) < 2.1$. The scour hole radius of the unsubmerged jet is less than that produced by a submerged jet for $F'_0 / (H/d) < 1$.

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Canada (NSERC) provided financial assistance for this project through a grant to the first writer. The writers are also thankful for the helpful comments of the reviewers.

Notation

The following symbols are used in this paper:

- $D$ = mean size of sand particles (50% of particles are finer);
- $d$ = diameter of jet at nozzle;
- $d'$ = diameter of jet as it impinges on bed;
- $F_0 =$ densimetric Froude number based on jet velocity at nozzle;
- $F_0'$ = densimetric Froude number based on jet velocity at original bed level;
- $g =$ acceleration due to gravity;
- $H =$ height of jet above sand bed;
- $R'$ = jet Reynolds number at original bed level;
- $r =$ distance from jet centerline;
- $\overline{r}^2 =$ correlation coefficient;
- $r_{0s} =$ radius of scour hole at asymptotic state;
- $U_0 =$ velocity of jet at nozzle;
- $U_0' =$ velocity of jet at level of sand bed;
- $\Delta \rho =$ difference in density between sand particles and eroding fluid;
- $\varepsilon =$ local scour depth at distance $r$ from jet centerline;
- $\varepsilon_{ms}' =$ maximum static scour depth at asymptotic state;
- $\varepsilon_{md}' =$ maximum dynamic scour depth at asymptotic state;
- $\nu =$ kinematic viscosity of eroding fluid; and
- $\rho =$ density of eroding fluid.

Subscripts

- * = equivalent scour produced by submerged jet.

References


