Estimation of inelastic displacement ratio for base-isolated structures

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Summary
This study develops a straightforward approximate method to estimate inelastic displacement ratio, \( C_1 \) for base-isolated structures subjected to near-fault and far-fault ground motions. Taking into account the inelastic behavior of isolator and superstructure, a 2 degrees of freedom model is employed. A total of 90 earthquake ground motions are selected and classified into different clusters according to the frequency content features of records represented by the peak ground acceleration to peak ground velocity ratio, \( A_p/V_p \). A parametric study is conducted, and effective factors in \( C_1 \) (i.e., fundamental vibration period of the superstructure, \( T_s \); postyield stiffness ratio of the superstructure, \( \alpha_s \); strength reduction ratio, \( R \); vibration period of the isolator, \( T_b \); strength of the isolator, \( Q \); ratio of superstructure mass to total mass of the system, \( \gamma_m \) ) are recognized. The results indicate that the practical range of \( C_1 \) values could be expected for base-isolated structures. Subsequently, effective parameters are included in simple predictive equations. Finally, the accuracy of the proposed approximate equations is evaluated and verified through error measurement, and comparisons are made in the analyses.

KEYWORDS
base-isolated structures, frequency content, inelastic displacement ratio, performance-based analysis

1 | INTRODUCTION

Nonlinear time history analysis (NTHA) is known as the most robust and accurate method for computing the seismic responses of structures where the main characteristics of earthquake records are completely considered. Owing to the expensive computational cost of NTHA, several simple methods have been proposed to estimate the nonlinear behavior of structures based on nonlinear static analysis.\(^1\)\(^-\)\(^5\) They include displacement coefficient method (DCM), capacity spectrum method, modal pushover analysis,\(^6\) and N2 method.\(^7\) Moreover, equivalent linearization methods are available as reported in Yaghmaei-Sabegh.\(^8\) In particular, DCM framework estimates the inelastic displacement of structures (target displacement) by applying a succession of coefficients upon the elastic spectral response of corresponding linear single degree of freedom (SDOF) system in its fundamental vibration mode. A key factor in the estimation of target displacement is inelastic displacement ratio (designated as coefficient \( C_1 \) in American documents\(^1\)\(^-\)\(^5\)). Inelastic displacement ratio was described for the first time by Veletsos and Newmark\(^9\) as the peak displacement response of an inelastic SDOF system divided by the elastic spectral response. Mainly, \( C_1 \) is based on \( R\cdot\mu\cdot T \) (strength reduction ratio-ductility ratio-vibration period) parameters. Furthermore, previous studies illustrated that structural parameters, site condition, and excitation characteristics are also effective in estimating \( C_1 \) in the case of fixed-base (FB) structures.\(^10\)\(^-\)\(^18\) More
recently, different frequency contents of near-fault (NF) pulse-like and far-fault (FF) ground motions have been recognized as crucial and need to be taken into account in $C_1$ predictive equations. It is noteworthy that the frequency content has commonly been characterized by pulse period ($T_p$) and the ratio of peak ground acceleration ($A_p$) to peak ground velocity ($V_p$).\(^{19-22}\)

On the other hand, base isolation has been used as a passive control strategy to improve seismic responses of structures. The main purpose of base isolation is the dissipation of input energy at the base and shifting the fundamental frequency of system away from the dominant frequencies of a seismic ground motion by using a flexible bearing. The design objective for buildings with seismic isolation is to achieve (fully) operational performance level. It means that superstructure in base-isolated (BI) buildings should remain elastic in the case of the design basis earthquake, but it could experience some minor damage under maximum considered earthquake. In other words, nonlinear behavior could be observed when BI structures experience near-source strong shaking.\(^{23}\) Palazzo and Petti\(^{24}\) recommended strength reduction ratios ranging between 1.5 and 3 based on the collapse spectrum of BI structures with different lateral resisting systems. Eventually, due to some rational perceptions, design and retrofitting codes (Eurocode\(^{25}\) and ASCE \(^{26}\) proposed strength reduction ratio ($R$) about 1 to 2 for BI structures, and inelastic displacement ratio is simply considered to be 1 ($C_1 = 1$).\(^{1,4}\)

It is commonly agreed that BI structures experience higher ductility demand compared with counterpart FB buildings with a specific strength reduction ratio.\(^{27-32}\) Sayani and Ryan\(^{33}\) generated constant ductility spectra for FB and BI buildings and compared the performance objectives of the 2 systems. They outlined that a general methodology could be extended to potentially fill a critical gap in the performance-based design of BI structures. Goda et al.\(^{34}\) developed a model to predict peak ductility demand of BI structures. Vassiliou et al.\(^{37}\) conducted a parametric study to investigate the inelastic displacement ratio of BI structures by using 2 degrees of freedom (2DOF) isolator-structure system. They observed an effective shortening of the displacement control region (where the “equal displacement” rule $\mu = R$ applies) of the inelastic displacement ratio spectrum for inelastic seismically isolated structures. Subsequently, they proposed a relationship to determine the strength reduction ratio of BI structures.\(^{35}\) Castaldo et al.\(^{36}\) presented seismic reliability-based relationships between the strength reduction ratio and the displacement ductility demand of nonlinear structural systems equipped with friction pendulum (FP) isolators. Yaghmaei-Sabegh et al.\(^{28}\) studied the inelastic displacement ratio of BI structures considering cyclic degradation and pinching in the hysteretic behavior of superstructure. They found that such hysteresis parameters amplify inelastic responses, especially in the case of short-period superstructures. They also observed a close correlation between $A_p/V_p$ ratio and $C_1$. The same conclusion about the impacts of $A_p/V_p$ ratio was reported by Dicleli and Buddaram\(^{37}\) to develop an equivalent linear analysis procedure for base isolation systems. Providakis\(^{38}\) demonstrated that BI structures can represent inelastic behavior to achieve a specified target displacement. Moreover, the inelastic behavior of superstructure under pulse-like NF ground motions at the maximum considered earthquake level was reported by Calugaru and Panagiotou.\(^{39}\) Cardone et al.\(^{40}\) investigated inelastic seismic response of seismically isolated RC frame buildings. They recommended a global strength reduction ratio ($R = 2$) for BI buildings featuring a weak-beam/strong-column inelastic mechanism.\(^{40}\) Some of the few works that studied the inelastic behavior of superstructure as well as their damage state are available in Cardone et al, Kilar and Koren, and Kilic et al.\(^{40-43}\) Kilar and Koren\(^{41-43}\) found that BI structures designed at the limit of elastic range might represent an inelastic response as a consequence of the superstructure asymmetries or the distribution of base isolation devices in the plan or bidirectional excitation.

From the aforementioned literature review, it seems to be essential to develop a reliable designing and retrofitting procedure for BI buildings considering the inelastic behavior of superstructure. Thus, this paper deals with the inelastic displacement ratio ($C_1$) of BI structures to develop new predictive equations. To this end, the research is conducted through the following phases. First, the aims and methodology of the study are presented, and then models of BI structures are developed based on specific criteria. Second, a wide range of earthquake records are selected and categorized according to representative groups. In the third step, several parameters that affect inelastic demand of BI structures are evaluated and 2 relations are proposed to estimate the inelastic displacement ratio for BI structures subjected to NF and FF earthquakes. Finally, errors and verification of the presented formulas are discussed from different aspects.

2 | OBJECTIVE AND METHODOLOGY OF RESEARCH

In recent decades, the popularity of SDOF systems for developing inelastic displacement ratio formulas has led to their widespread utilization in FB structures. With the aim of developing specific equations to estimate the inelastic
displacement ratio ($C_1$) for BI structures, an efficient methodology is employed by using a 2DOF simplified model. While providing facilities of extensive parametric studies, this model has been extensively used to investigate the performance of BI structures in recent years. Several parameters affect the inelastic responses of BI structures during seismic events. Therefore, numerous nonlinear time history analyses are preformed, and some constraints are considered to pace the analyses and prediction procedures. Bilinear Bouc-Wen model is used to characterize the inelastic behavior of isolator and superstructure. The first yielding in the superstructure pushover curve is assumed as a point of excursion into the nonlinear range (secondary slope of bilinear idealization). It has been clearly observed in previous studies that predicting the inelastic demand of structures is relatively correlated with the frequency content of earthquake excitation. Thus, earthquake records are carefully selected to highlight most features of NF and FF ground motions. $A_p/V_p$ ratio as a reliable parameter is used to characterize the frequency content of selected records. It is noteworthy that researchers have demonstrated the effects of $A_p/V_p$ ratio on structural responses such as structural damage, the inelastic displacement ratio of SDOF systems, the seismic response of BI structures, and the development of equivalent linearization model for base isolation systems. Meanwhile, some predictive equations have been proposed for $A_p/V_p$ ratio to facilitate the evaluation of seismic responses.

The present study discusses the findings for FP bearings unless explicitly stated otherwise. The models are simple shear-type elements including bilinear isolator and the superstructure hysteresis with no degradations. The results and the proposed equations are based on medium to high intensity levels of excitation. Specific isolator strength is assumed which, according to parametric study, can be used in the case of higher isolator strength conservatively. The strength and displacement capacity of the isolation system are also assumed to be adequate to insure its uninterrupted function. Thus, the failure of the isolators is not modeled in this study.

### 2.1 Modeling of two degrees of freedom base-isolated inelastic system

As illustrated in Figure 1, the BI structure is modeled by 2DOF system in this paper. This simplification could be justified based on the fact that the effects of higher modes on superstructure behavior are not noticeable. The well-known Bouc-Wen model is adopted to characterize the bilinear hysteretic response of isolator and superstructure in this study. The model can properly represent shear deformation of elastomeric (e.g., lead-rubber bearing) as well as FP bearings. A more complex model with velocity-dependent friction coefficient has not been taken into account for the sake of simplicity. The equations of motion for given 2DOF system can be presented as follows:

\[
\begin{align*}
    m_b \ddot{u}_b + c_b \dot{u}_b + k_b u_b + Q \dot{Z}_b - c_s \dot{u}_s - \alpha_s k_s u_s - (1 - \alpha_s) k_s u_{ys} Z_s &= -m_b \ddot{u}_g \\
    m_s \ddot{u}_s + c_s \dot{u}_s + \alpha_s k_s u_s + (1 - \alpha_s) k_s u_{ys} Z_s &= -m_s (\ddot{u}_g + \ddot{u}_b) \\
    Q &= \mu_f (m_b + m_s) g
\end{align*}
\]

where $u_s$ is superstructure displacement relative to the base and $u_b$ is base displacement relative to the ground ($b$ and $s$ stand for base isolation system and superstructure, respectively). Different parameters of Equations 1 and 2 are
illustrated in Figure 1. \( Q \) is the strength of isolator based on specific friction coefficient \((\mu_f)\) (Equation 3). \( m, c, \) and \( k \) are mass, damping, and stiffness values, respectively, and \( \alpha_s \) is the ratio of postyield to initial stiffness of superstructure. Isolation period \( T_b = 2\pi \sqrt{m_s + m_b/k_b} \) is defined based on postyield stiffness of isolator \( k_y \). \( z \) can be derived based on the following differential equation:

\[
\dot{z}_{i=0} = \frac{1}{\mu_f} \left\{ \ddot{u}_i - (\beta_i |\dot{u}_i|^{\gamma_i} - \gamma_i |\dot{u}_i|^n) \right\}
\]

\( u_{yb} \) is the yield displacement of the isolator bilinear model, which is assigned a very low value close to 0 in this study \((u_{yb} \approx 0.25 \text{ mm})\). \( u_{ys} \) is the yield displacement of the isolated superstructure. \( \beta, \gamma, \) and \( n \) are shape parameters. \( \beta \) and \( \gamma \) are assumed 0.5, and \( n \) is set equal to 2, which adjusts the smoothness of the transition from the first to the second slope of the response envelope.\(^{50-53}\)

### 2.2 Considered parameters

The main parameters affecting the inelastic response of BI structures are listed in Table 1. Strength reduction ratio \((R)\) is defined as the ratio of a minimum strength required to maintain the response of a superstructure in the elastic range \( F_{els} \) to the superstructure yield strength \( F_{ys} \). Basically, a lower strength reduction ratio for BI structures is recommended in the literature\(^{25,26,35}\); therefore, \( R \) values are limited to 1 to 3. The period of vibration for superstructure is considered between 0 and 4 seconds. However, due to the robustness of analyses, greater period steps are reasonably assumed in the higher period region of spectrum based on lower coefficient of variation (COV) of \( C_1 \) values in this period region.\(^{19,28}\)

The postyield stiffness ratio in the global force deformation of a structure has been known as an essential parameter, which affects the inelastic displacement ratio\(^{16,19,27}\); therefore, it is assumed in the range of 0 to 0.2 in this investigation. Superstructure damping ratio \((\xi_s)\) is set equal to 2%. Three distinct mass ratios \((\gamma_m = m_s/(m_s + m_b))\) equal to 0.5, 0.8, and 0.95 are taken into account for low-,- medium-, and high-rise (light to heavy) superstructures, respectively.

There are several types of base isolation devices with various characteristics. They are classified into 2 general groups of (i) sliding bearings such as pure friction, FP, and multiple FP bearings, and (ii) elastomeric bearings, for example, high and low damping rubbers and lead rubber bearings. Modeling FP bearing based on a bilinear hysteresis associated with \( \xi_b = 0\% \) is adopted herein. Friction coefficient values were commonly reported between 0.05 and 0.3.\(^{29,54-56}\) The friction coefficient of isolators is set to be 0.05. In other words, the friction coefficient of the isolation system \( \mu_f \) can first be normalized with respect to the ground motion inertia force demand \( A_p/g \) following the dimensional analysis results as presented by Vassiliou et al.\(^{27}\) Therefore, the response of a superstructure would be identical for a specific \( \mu_f/(A_p/g) \). More details about the effects of friction coefficient are provided in Section 4. The vibration period of isolators is supposed to vary between 1.5 and 4 seconds. Accordingly, the form of predictive equations generated based on the data trends and statistical procedure is used to determine the coefficients of the formulas. The steps for the evaluation of dynamic responses and data processing to estimate the inelastic displacement ratio of the BI buildings are illustrated in Figure 2.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Symbol</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superstructure</td>
<td>R</td>
<td>Strength reduction ratio</td>
<td>([1.5, 2, 2.5, 3])</td>
</tr>
<tr>
<td></td>
<td>( T_s )</td>
<td>Vibration period of structure</td>
<td>([0.1:0.1:1, 1.2:0.2:2, 2.25:0.25:4])</td>
</tr>
<tr>
<td></td>
<td>( \alpha_s )</td>
<td>Postyield stiffness ratio of structure</td>
<td>([0, 0.02, 0.05, 0.1, 0.2])</td>
</tr>
<tr>
<td></td>
<td>( \xi_s )</td>
<td>Damping ratio of structure</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>( \gamma_m )</td>
<td>Mass ratio</td>
<td>([0.5, 0.8, 0.95])</td>
</tr>
<tr>
<td>Isolator</td>
<td>( T_b )</td>
<td>Vibration period of isolator</td>
<td>([1.5, 2.5, 3, 4])</td>
</tr>
<tr>
<td></td>
<td>( \xi_b )</td>
<td>Damping ratio of isolator</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( \mu_f = F_0 )</td>
<td>Friction coefficient or normalized yield force of isolator</td>
<td>0.05</td>
</tr>
<tr>
<td>Excitation</td>
<td>( A_p/V_p )</td>
<td>Peak acceleration/peak velocity (frequency content)</td>
<td>See Tables 2 and 3</td>
</tr>
</tbody>
</table>
EXCITATION CHARACTERISTICS

3.1 Selected ground motion records

Two sets of seismic excitations, namely, NF and FF earthquakes, were obtained from the Pacific Earthquake Engineering Research Center database (NGAWEST2, 2016). Characteristics of NF ground motions may differ from those of FF motions because of 2 distinct specifications: directivity and fling step. Considerable energy imposed on structures in a short duration under NF excitations including directivity effect could be critical. Subset 1 in this study contains 45 FF nonpulse-like ground motions classified into 3 groups with different \( A_p/V_p \) ratios, that is, low \( A_p/V_p \) ratios (\( A_p/V_p < 8 \) Hz), intermediate \( A_p/V_p \) ratios (\( 8 \) Hz < \( A_p/V_p < 12 \) Hz), and high \( A_p/V_p \) ratios (\( A_p/V_p > 12 \) Hz; Table 2). Similarly, there are 45 NF pulse-like motions in subset 2 classified into 3 groups, that is, low \( A_p/V_p \) ratios (\( A_p/V_p < 5 \) Hz), intermediate \( A_p/V_p \) ratios (\( 5 \) Hz < \( A_p/V_p < 8 \) Hz), and high \( A_p/V_p \) ratios (\( A_p/V_p > 8 \) Hz; Table 3). For each event with 2 horizontal components, the component with the maximum response of pseudo-acceleration in period range 0.5T_D to 1.25T_M was selected based on the recommendation of ASCE7-10. T_D and T_M are effective periods of the isolated structure at design and maximum displacement, respectively. Other specifications of the selected records are (1) \( M \geq 5 \); (2) different fault types (ie, strike-slip, reverse, oblique, and normal); (3) closest site-to-fault distance less than 20 km and PGV > 30 (cm/s) for NF earthquakes; (4) soil types B, C, and D based
Far-fault ground motions used in this study

<table>
<thead>
<tr>
<th>EQ No</th>
<th>Event</th>
<th>Year</th>
<th>Station</th>
<th>$M_w$</th>
<th>$R_{rup}$ (km)</th>
<th>$V_{30}$ (m/s)</th>
<th>Com (deg)</th>
<th>PGA (g)</th>
<th>PGV (cm/s)</th>
<th>PGD (cm)</th>
<th>$A_p/V_p$ (Hz)</th>
<th>$T_g$ (s)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>Tabas, Iran</td>
<td>1978</td>
<td>Kashmar</td>
<td>7.36</td>
<td>194.55</td>
<td>280.26</td>
<td>T1</td>
<td>0.036</td>
<td>9.64</td>
<td>4.37</td>
<td>3.66</td>
<td>1.83</td>
</tr>
<tr>
<td>2</td>
<td>Landers</td>
<td>1992</td>
<td>Anaheim-W Ball Rd</td>
<td>7.28</td>
<td>144.9</td>
<td>269.29</td>
<td>0</td>
<td>0.047</td>
<td>12.48</td>
<td>10</td>
<td>3.69</td>
<td>3.52</td>
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<tr>
<td>3</td>
<td>Duzce</td>
<td>1999</td>
<td>Aslan R</td>
<td>7.18</td>
<td>130.81</td>
<td>336.98</td>
<td>90</td>
<td>0.015</td>
<td>4.2</td>
<td>3.64</td>
<td>3.50</td>
<td>7.18</td>
</tr>
<tr>
<td>4</td>
<td>Kobe, Japan</td>
<td>1995</td>
<td>OSAJ</td>
<td>6.9</td>
<td>21.35</td>
<td>256</td>
<td>0</td>
<td>0.082</td>
<td>19.24</td>
<td>8.84</td>
<td>4.18</td>
<td>1.12</td>
</tr>
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<td>5</td>
<td>Cape Mendocino</td>
<td>1992</td>
<td>Fortuna-Fortuna Blvd</td>
<td>7.01</td>
<td>20</td>
<td>457.06</td>
<td>90</td>
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<td>26.433</td>
<td>9.94</td>
<td>4.19</td>
<td>3.2</td>
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<tr>
<td>6</td>
<td>Borrego Mtn</td>
<td>1968</td>
<td>El Centro Array #9</td>
<td>6.5</td>
<td>45</td>
<td>213.44</td>
<td>180</td>
<td>0.132</td>
<td>26.7</td>
<td>14.6</td>
<td>4.85</td>
<td>1.87</td>
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<tr>
<td>7</td>
<td>Kocaeli, Turkey</td>
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<td>Bursa Sivil</td>
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<td>612.78</td>
<td>90</td>
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<tr>
<td>8</td>
<td>Kobe, Japan</td>
<td>1995</td>
<td>Kakogawa</td>
<td>6.9</td>
<td>22.5</td>
<td>312</td>
<td>90</td>
<td>0.324</td>
<td>26.87</td>
<td>8.85</td>
<td>11.82</td>
<td>1.78</td>
</tr>
</tbody>
</table>

(Continues)
on ASCE7-100; and (5) low-cut filter is smaller than or equal to 0.1 Hz. Mean acceleration response spectra of the 2 sets of NF and FF records are shown in Figure 3. The records were scaled to a maximum acceleration of 0.35 g.

4 | EFFECTIVE PARAMETERS

Because the effects of different characteristics of ground motions on inelastic displacement ratio of FB structures are available in,22 we merely highlighted the effects of structural system and isolator properties on the inelastic displacement demand of BI structures herein.

4.1 | Effects of strength reduction ratio (R)

To illustrate the effects of strength reduction ratio on the inelastic displacement ratio of BI structures, the mean and COV of $C_1$ values were obtained for $\alpha_s = 0.05$, $\gamma_m = 0.8$, $T_b = 4$ seconds based on the both sets of the records and are presented in Figure 4. It can be seen that a greater $R$ value results in a higher $C_1$ specifically for a lower superstructure period. An ascending trend is observed in inelastic displacement spectra by decreasing $T_s$; however, a different trend can be seen for low to intermediate $A_p/V_p$ ratios, especially in the case of NF ground motions. The equal displacement region of spectrum ($C_1 = 1$) is initiated from lower $T_s$ providing that the superstructure develops a greater lateral strength (small $R$). Mean values of $C_1$ for BI structures under NF records are generally greater in comparison with those in FF records for each band of $A_p/V_p$ ratios. This observation can be attributed to the pulse-like behavior of NF ground motions and also the lower bands of $A_p/V_p$ ratios selected for NF records according to Tables 2 and 3. For example, according to Figure 4A and B for $R = 3$ and low $A_p/V_p$ ratios, NF records with $A_p/V_p < 5$ Hz generate higher $C_1$ values compared with the ones under FF records with $A_p/V_p < 8$ Hz. However, it can be seen that the $C_1$ values for FF ($A_p/V_p < 8$ Hz) correspond to those of NF ($5$ Hz $< A_p/V_p < 8$ Hz). In other words, NF records are more probable to produce lower $A_p/V_p$ ratios ($A_p/V_p < 5$ Hz). Accordingly, $A_p/V_p$ ratio is a preferable parameter to quantify record to record variability. Based on Figure 4C and D, the superstructures with a higher strength reduction ratio produce greater dispersion. The same situation is observed for records with high $A_p/V_p$ ratios, which can be due to higher variation of $A_p/V_p$ ratios in this group of records.

In Figure 5, the mean $C_1$ values are plotted against the strength reduction ratio of superstructure with $T_s = 1$ second. Two different isolator periods and mass ratios were considered along with the postyield stiffness ratio of 0.02. It is evident that the inelastic displacement ratio rises by increasing $R$. The growth is higher for pulse-like NF records with forward directivity effect. The variation of $C_1$ versus $R$ is more considerable for ground motions with low values of $A_p/V_p$ ratios.

<table>
<thead>
<tr>
<th>Table 2 (Continued)</th>
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<tr>
<td>$A_p/V_p &lt; 8$ Hz</td>
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<tr>
<td>EQ No</td>
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<tr>
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</tr>
<tr>
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Table 3 Near-fault pulse-like ground motions used in this study

<table>
<thead>
<tr>
<th>EQ No</th>
<th>Event</th>
<th>Year</th>
<th>Station</th>
<th>$M_w$</th>
<th>$R_{rup}$ (km)</th>
<th>$V_{50}$ (m/s)</th>
<th>Com (deg)</th>
<th>PGA (g)</th>
<th>PGV (cm/s)</th>
<th>PGD (cm)</th>
<th>$A_p/V_p$ (Hz)</th>
<th>$T_p$ (s)</th>
<th>$T_g$ (s)</th>
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<tbody>
<tr>
<td>1</td>
<td>Chi-Chi, Taiwan</td>
<td>1999</td>
<td>TCU068</td>
<td>7.62</td>
<td>0.32</td>
<td>487.34</td>
<td>E</td>
<td>0.511</td>
<td>249.6</td>
<td>294.13</td>
<td>2.01</td>
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<td>59</td>
<td>2.32</td>
<td>5.38</td>
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<tr>
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<td>2002</td>
<td>TAPS PS #10</td>
<td>7.9</td>
<td>2.74</td>
<td>329.4</td>
<td>47</td>
<td>0.332</td>
<td>115.7</td>
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8 Hz < $A_p/V_p$

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(Continues)
Similarly, a greater increment of $C_1$ with respect to $R$ is resulted for isolators with a higher period of vibration. It is also observed that $C_1$ trend in relation to $R$ takes polynomial form in the case of higher mass ratios.

### 4.2 Effects of postyield stiffness ratio of structure ($\alpha_s$)

In Figures 6 and 7, the calculated mean and COV of $C_1$ values are plotted for system characteristics $R = 2 - \gamma_m = 0.8 - T_b = 3$ seconds to investigate the effects of the postyield stiffness ratio of the superstructure. It is confirmed that higher $\alpha_s$ significantly reduces $C_1$, specifically in the short period region of spectrum.

As an interesting finding, $C_1$ in BI structures takes acceptable values from engineering practice outlook in case of $\alpha_s > 0.1$. It means that, for low period superstructures with $\alpha_s > 0.1$, the ductility demand may not exceed the ductility capacity of the superstructure. Nonetheless, an extensive damage or collapse may occur in the case of superstructures with a low $\alpha_s$ value, especially under strong earthquakes with low $A_p/V_p$ ratios. On the other hand, the mean $C_1$ is monotonously growing as the period of vibration decreases for superstructures with no postyield stiffness ratio; however, the trend of $C_1$ spectrum changes following the introduction of $\alpha_s$, where $C_1$ trend flattens in the low period region of spectrum.

**TABLE 3** (Continued)

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**FIGURE 3** Mean response spectra of scaled records: (A) near-fault (NF) and (B) far-fault (FF) [Colour figure can be viewed at wileyonlinelibrary.com]
A different frequency content of ground motions characterized by $A_p/V_p$ ratio is obviously effective in the trend of $C_1$ spectrum. It can also be concluded that $\alpha_s$ influences $C_1$ variation pattern where it is mostly related to the period of superstructure. Sensitivity of $C_1$ values to postyield stiffness ratio decreases for higher $A_p/V_p$ ratios. Dispersion results revealed that record-to-record variability of the responses is remarkably reduced with an increase in $\alpha_s$ values, specifically in the case of short period range. The distribution of COV values is between 0.1 and 0.5, and lower $A_p/V_p$ ratios generate a smaller COV. Moreover, it can be seen that the sensitivity of inelastic displacement ratio to the frequency content of ground motion is lower for higher $\alpha_s$ values, and the COV of $C_1$ is computed around 0.2.

The plots in Figure 8 illustrate the variation of $C_1$ versus $\alpha_s$ for 3 different period regions and the system with the above-specified characteristics. It is observed that the slope of the change is reduced by increasing $A_p/V_p$ ratio, and a sharper downfall develops for $\alpha_s \leq 0.1$. The reduction trends of $C_1$ values are similar for both of the FF and NF records. According to Figures 6 to 8, it can be seen that a critical rise occurs in $C_1$ spectra for $T_s < 1$ second, and the effect of $\alpha_s$ becomes more highlighted in this region of the spectrum. In addition, $C_1$ values for $T_s = 1$ second are between 1 and 3, and for $T_s > 1$ second, $C_1$ tends to be 1.

### 4.3 Effects of mass ratio ($\gamma_m$)

In this section, the effects of mass ratio on the variation of $C_1$ are analyzed and reported. The mean and COV of $C_1$ for system characteristics $R = 2$, $\alpha_s = 0.05$, and $T_b = 3$ seconds in different levels of excitations and mass ratios are presented in Figure 9. Significant differences are observable in the trend and smoothness of $C_1$ spectrum with respect to various mass ratios where a higher mass of superstructure makes the spectrum smoother. Moreover, higher $C_1$ ratios should be expected for greater mass ratios, especially in the medium period region of the spectrum. It should be noted that by increasing $\gamma_m$, the sensitivity of the inelastic displacement ratio to that of $A_p/V_p$ is reduced. As a result, the inelastic
FIGURE 5  Variation of $C_1$ against strength reduction ratio ($R$) for $\alpha_s = 0.02$, $T_s = 1$ second (NF: thick line, FF: thin line) [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 6  Mean and coefficient of variation (COV) of $C_1$ values obtained for $R = 2$, $\gamma_m = 0.8$, and $T_b = 3$ seconds considering different postyield stiffness ratio ($\alpha_s$) under far-fault (FF) ground motions: (A and D) low $A_p/V_p$, (B and E) intermediate $A_p/V_p$, and (C and F) high $A_p/V_p$ [Colour figure can be viewed at wileyonlinelibrary.com]
displacement demand of BI structures with heavy superstructures is less sensitive to the frequency content of ground motion. It is also observed that lighter superstructures experience lower inelastic displacement ratios in the case of intermediate to high $A_p/V_p$ ratios. Thus, $C_1$ is influenced by superstructure mass. Figure 9D to F reveals that dispersion of $C_1$ values is remarkably lower for high mass ratios. Higher dispersion of $C_1$ is observed for light superstructures by decreasing $A_p/V_p$ ratio, whereas, for heavy ones, this behavior is vice versa. It is notable that higher values of COV for $\gamma_m = 0.95$ and high $A_p/V_p$ ratios are due to the higher variation of $A_p/V_p$ ratios for the selected records in the bin. The variation of $C_1$ against mass ratio is depicted in Figure 10. According to this figure, the influence of $\gamma_m$ variation becomes more pronounced by rising $A_p/V_p$ ratios for both FF and NF records. Besides, it can be seen that $C_1$ values are not sensitive to mass ratio in the case of very low $A_p/V_p$ ratios under NF records.

### 4.4 Effects of period of isolator ($T_b$)

Figure 11 shows mean and COV of $C_1$ for 3 different isolator periods ($T_b = 1.5, 3, 4$ s) under FF and NF ground motions when other parameters are fixed as $R = 2, \alpha_s = 0.05$, and $\gamma_m = 0.8$. It is observed that a longer period of
isolation system would result in higher values of $C_1$ in the case of low $A_p/V_p$ ratios. In addition, it can be seen that the inelastic displacement demand of BI structures with a longer isolator period is more sensitive to the frequency content of the excitation. It is noteworthy that the equal displacement region possibly starts from $T_s \geq T_b$. The dispersion of $C_1$ values is almost the same for different isolation periods; however, an increase is observed for intermediate to high $A_p/V_p$ ratios in the case of longer isolation periods. According to Figure 12, it can be concluded that $C_1$ tends to increase when the difference between $T_s$ and $T_b$ is greater particularly under excitations with a high potential damage as visible in Figure 12A. Therefore, $C_1$ spectrum depends on $T_b$, especially in the cases of low and intermediate $A_p/V_p$ ratios.

### 4.5 | Effects of isolator strength ($Q$)

The effects of isolator strength are discussed in more details herein. For this purpose, a nondimensional strength ratio $Q/[(m_s + m_b)A_p] = \mu_f/(A_p/g)$, proposed by Vassiliou et al.,27 is used in this study. Based on Tsiavos et al.,35 an applicable earthquake $A_p$ along with a range of bearing friction coefficients ($\mu_f$) that correspond to the nondimensional strength
ratio $\mu/(A_p/g)$ could be considered. According to Table 5 in Tsiavos et al.$^{35}$ $\mu/(A_p/g) < 0.075$ was reported for extremely strong earthquakes with $A_p > 1.5$ g. Consequently, $\mu/(A_p/g) > 0.075$ is applied for a comparative study in this section. Moreover, the assumed friction coefficient in this study is consistent with the reported range of this parameter (i.e., between 0.04 and 0.12) in Tsiavos et al.$^{35}$

Figure 13 illustrates the mean values of $C_1$ and COV ($C_1$) for $R = 2$, $\gamma_m = 0.8$, and $T_b = 3$ seconds in different levels of $\mu/(A_p/g) = [0.075, 0.143, 0.3, 0.45]$ under NF excitations. For the sake of brevity, only the results for NF ground motions are presented. However, a similar trend was observed in FF records. It can be seen that for a specific intensity, an isolator with a higher strength demonstrates lower $C_1$ values. From Figure 13A and B, it is observed that $C_1$ spectra present identical values and trends for $\mu/(A_p/g) = 0.075$ and 0.143, specifically under excitations with low and intermediate $A_p/V_p$ ratios. Nevertheless, $C_1$ values are smaller for higher $\mu/(A_p/g)$ ratios. Therefore, the value of $\mu/(A_p/g) = 0.143$, which represents medium to high intensity ground motions, can be considered as a conservative estimation. Coefficient of variation ($C_1$) values are similar for different levels of $\mu/(A_p/g)$ and vary between 0.2 to 0.4. Generally, the results indicate that the behavior can be categorized into 2 main levels of $\mu/(A_p/g)$: lower and higher than 0.3). Moreover, the responses of BI structures with a strong isolator $\mu/(A_p/g) > 0.3$ resemble those in FB ones (lower $C_1$) as reported in Tsiavos et al.$^{35}$ too.
4.6 Effects of $A_p/V_p$ ratio

To depict the effects of excitation characteristics on inelastic displacement demand of BI structures, all of the selected ground motions in Tables 2 and 3 are considered. Figure 14 illustrates the variation of $C_1$ based on $A_p/V_p$ ratios for

FIGURE 13  Mean and coefficient of variation (COV) of $C_1$ values obtained for $R = 2$, $\gamma_m = 0.8$, and $T_s = 3$ seconds considering different $\mu_f/(A_p/g)$ under NF ground motions: (A and D) low $A_p/V_p$, (B and E) intermediate $A_p/V_p$, and (C and F) high $A_p/V_p$. [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 14  Functional variation of $C_1$ against $A_p/V_p$ ratio for $R = 2$ and $T_s = 0.5$ second. [Colour figure can be viewed at wileyonlinelibrary.com]
R = 2 and \( T_s = 2 \) seconds and considered variable parameters used in the analyses. In this figure, the plots describe the differences among various values selected for \( \gamma_m \), \( T_b \), and \( \alpha_s \). Figure 14A to D and Figure 14E and F compare the effect of postyield stiffness ratio. Meanwhile, compared with Figure 14C, D, G, and H, Figure 14A, B, E, and F depict the effect of mass ratio. The effects of \( T_b \) are illustrated in Figure 14A, C, E, and G and Figure 14B, D, F, and H. It is observed that power function form \( Y = aX^b \) is governed to represent \( C_1 \) trend based on \( A_p/V_p \) ratio as reported in Durucan and Dicleli. The comparison of Figure 14K, L, O, and P with the rest of the plots points out to the fact that superstructures with higher masses and postyield stiffness ratios represent the least slopes of variations against \( A_p/V_p \) ratios. It is noteworthy that the effects of \( A_p/V_p \) ratio on \( C_1 \) values are more pronounced for a higher period of the isolator. As a result, a high superstructure mass, postyield stiffness ratio, and low isolator period generate an inelastic displacement with no sensitivity to the frequency content of excitation and vice versa. Therefore, the effects of \( A_p/V_p \) factor are strongly influenced by \( \gamma_m \), \( T_b \), and \( \alpha_s \).

5 | PREVIOUS \( C_1 \) EQUATIONS

A number of equations representing \( C_1 \) have been proposed in the literature for FB structures. The available equations in design guidelines like FEMA 356[4] and ASCE/SEI 41-131 and those proposed by Ruiz-García and Miranda,14 Iervolino et al,21 and Durucan and Dicleli22 are plotted in this section together with the exact data obtained through the analyses of BI structures for comparison. Among these equations, Ramirez et al13 and Chopra and Chintanapakdee16 have considered postyield stiffness ratio effects in their suggested \( C_1 \) equations. Figure 15A compares the exact data of FF analyses for isolated system \((R = 2, \alpha_s = 0, \gamma_m = 0.8, T_b = 3 \text{ s})\) along with the \( C_1 \) spectrum obtained from FEMA 356,4 Ramirez et al,13 and Chopra and Chintanapakdee16 equations. It is observed that these equations often underestimate \( C_1 \) values and cannot capture the variations in responses. Based on Figure 15B, the exact data of the soil class C (NF) analyses for BI structures are plotted together with the obtained values of the equations given by ASCE/SEI 41-131 (Equation 5) and Ruiz-Garcia and Miranda.14 In Equation 5, \( R \) is the strength reduction ratio, \( T_s \) is the effective fundamental period of structure vibration, and \( a \) is a constant depending on site conditions. These equations also underestimate \( C_1 \) values for BI structures.

Recently, pulse period \((T_p)\) and peak ground acceleration to peak ground velocity \((A_p/V_p)\) have been considered as 2 key excitation features to better estimate the inelastic demand of structures. Figure 16A illustrates the obtained values from Iervolino et al21 equation along with the exact data of NF analyses for BI structures \((R = 2, \alpha_s = 0, \gamma_m = 0.8, T_b = 3 \text{ s})\). The horizontal axis in this figure is normalized by \( T_p \). It is observed that the normalization could reduce the dispersion of data, and \( T_p \) can be interpreted as an effective proxy representing the frequency content of the excitation in estimating the inelastic displacement ratio of BI structures. Nevertheless, the suggested equation by Iervolino et al21 underestimates the \( C_1 \) values and may need some modifications.

Durucan and Dicleli22 proposed Equation 6 for \( C_1 \) estimation of FB structures, which includes \( A_p/V_p \) ratio of seismic excitation. The constants \( x_1, x_2, x_3, \) and \( x_4 \) in Equation 6 were given as 1, 0.8, 1.5, and 4/3, respectively, for FF case and

![Figure 15](https://example.com/figure15.png)

**FIGURE 15** Calculated \( C_1 \) values of base-isolated (BI) structure \((R = 2, \alpha_s = 0, \gamma_m = 0.8, T_b = 3 \text{ s})\) versus approximated \( C_1 \) of FS \((R = 2)\): (A) far-fault (FF) and (B) soil class C near-fault (NF) [Colour figure can be viewed at wileyonlinelibrary.com]
6/5, 0.6, 1.1, and 1.6 for NF case. Figure 16B and C show the $C_1$ spectra obtained from Equation 6 and the exact data calculated from NTHA of BI structures under NF and FF records against normalized horizontal axis by $(2\pi/A_p/V_p)$. Similarly, the dispersion of $C_1$ values due to normalization decreases; however, it should be noted that Equation 6 underestimates $C_1$ values and needs some modifications for BI buildings.

$$C_1 = 1 + \frac{R-1}{aT_e}$$  \hspace{1cm} (5)

$$C_1 = 1 + \frac{(R-1)^{1/3}}{x_2T_2\left(A_p/V_p\right)}$$  \hspace{1cm} (6)

A few equations and figurative $R$-$\mu$-$T$ relations are available for BI buildings in which $C_1$ ratio is not directly calculated.\textsuperscript{35,36} The equation proposed by Tsiavos et al.\textsuperscript{35} for the calculation of strength reduction ratio ($R$) in the case of BI structures is rearranged to estimate $C_1$ as follows:

$$C_1 = \frac{T_b + c_Q(R-1) + 1}{c_aT_a} \quad T_b > T_a$$  \hspace{1cm} (7)

$$C_1 = \frac{T_b + c_Q(R-1) + 1}{c_aT_s} \quad T_a < T_s < T_c$$  \hspace{1cm} (8)

$$C_1 = 1 \quad T_s > T_c$$  \hspace{1cm} (9)

where $T_b$ and $T_s$ are isolator and superstructure vibration periods, respectively. $c_a$ and $c_Q$ are defined based on superstructure postyield stiffness ratio ($\alpha_s$) and $Q/[(m_s + m_b)A_p]$ ratio, respectively. $T_a$ and $T_c$ can also be calculated according to $T_b$, $c_a$, and $c_Q$. More details can be found in Tsiavos et al.\textsuperscript{35} For FP bearing, $Q/[(m_s + m_b)A_p] = \mu_p(A_p/g) = 0.143$ is used in this paper. It should be mentioned that the above relations were only presented for mass ratio $\gamma_m = 0.9$, which was identified as a conservative case.\textsuperscript{35} Figure 17 compares the results of the analyses for NF records considering $T_b = 3$ seconds, $R = 2$, $\alpha_s = [0.05, 0.1]$, and $\gamma_m = [0.8, 0.95]$ with the prediction of $C_1$ proposed by Tsiavos et al.\textsuperscript{35}

Generally, it can be seen that the proposed equations by Tsiavos et al.\textsuperscript{35} can effectively predict $C_1$ in the long period region of the spectrum. However, for medium to low period superstructures, the equations underestimate $C_1$ values. Moreover, Equations 7 to 9 were provided to take into account the effects of isolator period ($T_b$), postyield stiffness ratio ($\alpha_s$), and different levels of normalized isolator strength $Q/[(m_s + m_b)A_p]$ specifically to predict $R$. In other words, to our
best knowledge, the effects of the frequency content of excitation and mass ratio ($\gamma_m$) have not been considered in the literature.

6 | PROPOSED EQUATION

6.1 | Equation development

Performance-based assessment of structures under seismic events according to DCM has commonly been adopted for FB structures. It is intuitively appealing to propose a simplified model to estimate $C_1$ for BI structures (Figure 2). Due to special characteristics of NF ground motions, the records are divided into 2 groups of NF and FF with 3 different bins of $A_p/V_p$ ratios. Accordingly, the derivation of the equations proposed in this study is based on direct statistical formulation of the functional variation of $C_1$ with the abovementioned parameters.

Different forms of equations were primarily evaluated regarding the investigations in Section 4. Because there are many distinct parameters with different sources of effects, adjusting all of them in a unique form might not be sensible. Therefore, taking advantage of simple forms of Equations 5 and 6, $C_1$ equation suggested by ASCE41-13[1] is modified to be used for BI structures. The following basic form of the equation that properly describes the trends of $C_1$ spectra under different conditions is proposed:

$$C_1 = 1 + \frac{(R-1)^a}{b(T_s^2 + c)\left(\frac{A_p}{V_p}\right)^d}$$  \hspace{1cm} (10)
where \( a, b, c, \) and \( d \) are coefficients to be determined via nonlinear multivariate regression analysis by using the \( C_1 \) data obtained from NTHA. Nonlinear regression analysis based on the Gauss-Newton algorithm along with ‘Levenberg-Marquardt’ optimization procedure\(^{61}\) is used. This minimizes the sum of the squared differences of the exact responses from their fitted values. Based on Table 1, 496,800 \((90 \times 23 \times 4 \times 4 \times 5 \times 3 = 496,800)\) NTHA is performed to obtain the seismic response of 2DOF system considering bilinear hysteretic behavior. It is noteworthy that the data related to \( \alpha_s = 0 \) are not used in regression process due to the high dispersion and impractical range of calculated \( C_1 \) from an engineering point of view. However, resultant prediction is observed to be sufficient in the case of \( \alpha_s = 0 \).

The evaluation of effective parameters (in Section 4) leads to some sensible relationships between the parameters of considered dynamic system and coefficients of the proposed form of equation. Initially, we performed a 2-step regression analysis, which means first, the coefficients of Equation 10 are independently computed for each combination of variable parameters \((\gamma_m, T_b, \) and \( \alpha_s)\), and second, the variation of coefficients with respect to variable parameters is studied. The same procedure was employed by Hatzigeorgiou and Beskos\(^{17}\) to propose an equation for \( C_1 \) under repeated seismic events.

Based on sensitivity analysis with respect to \( \gamma_m, T_b, \) and \( \alpha_s \), the following relations are recommended (Equations 11-14). The form of equations expresses polynomial relations between the variable parameters and the coefficients of basic \( C_1 \) equation, which is consistent with the effects of parameters as discussed in Section 4. The accuracy of the proposed relations for the estimation of the coefficients is illustrated in Figure 18. It is obvious that there is a good agreement between predicted coefficients and the values obtained from the first step of the regression.

\[
a(T_b, \gamma_m) = a_1 + a_2 T_b + a_3 \gamma_m + a_4 \gamma_m^2
\]

\[
b(\alpha_s, T_b, \gamma_m) = b_1 + \frac{b_2 \alpha_s}{T_b} + b_3 T_b \frac{\alpha_s}{T_b} + b_4 \gamma_m + b_5 \gamma_m^2
\]

\[
c(\alpha_s, T_b, \gamma_m) = c_1 + c_2 \alpha_s + (c_3 \alpha_s) T_b + c_4 \gamma_m
\]

\[
d(T_b, \gamma_m) = d_1 + d_2 \frac{T_b}{\gamma_m} + d_3 T_b^2 + d_4 \gamma_m + d_5 \gamma_m^2
\]

Finally, Equations 11 to 14 are substituted in Equation 10, and 1 step regression analysis is performed. Table 4 represents the coefficients for FF and NF cases and the square of correlation coefficient \((R^2)\), which is calculated as

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**FIGURE 18** Accuracy of predicted coefficients for (A) far-fault (FF) and (B) near-fault (NF) records [Colour figure can be viewed at wileyonlinelibrary.com]
0.877 and 0.9057 for FF and NF records, respectively. It is noteworthy that using the coefficients of Table 4, $C_1$ values for $\alpha_s = 0$ can be predicted properly for $T_s \geq 0.2$ second. However, data dispersion for the case of $\alpha_s = 0$ rises, where $R^2$ values are obtained to be 0.7155 and 0.812 for FF and NF records, respectively.
6.2 Error measurement and verification

The error measures defined in FEMA 440 recommendation are employed to evaluate accuracy of the proposed $C_1$ equations for BI structures under NF and FF ground motions. Sample mean error ($E_{TR}$) and the standard deviation of the error ($\sigma_{TR}$) are determined as

![Figure 20](wileyonlinelibrary.com) Sample mean error for all of the data considered in the regression process: (A) far-fault (FF) and (B) near-fault (NF) [Colour figure can be viewed at wileyonlinelibrary.com]

![Figure 21](wileyonlinelibrary.com) Exact values of $C_1$ versus calculated $C_1$ by using the proposed equation for $R = 2$, $\alpha_s = 0.05$, and $T_b = 4$ seconds: (A and B) $\gamma_m = 0.8$ and (B and C) $\gamma_m = 0.95$ [Colour figure can be viewed at wileyonlinelibrary.com]
where $n$ is the number of records. These error measures are computed for each period of vibration and each level of relative lateral strength ($R$). Figure 19 shows error measure plots for 2 different postyield stiffness ratios ($\alpha_s = 0.02, 0.2$), isolator periods ($T_b = 1.5, 3$ s), and mass ratios ($\gamma_m = 0.5, 0.95$). It can be seen that mean error, in general, tends to be greater than 1, which means that the proposed equations overestimate the peak inelastic displacement demand. This is particularly true as the period of vibration of superstructure decreases. It is also observed that as the strength reduction ratio increases, the mean error grows. Mainly, higher strength reduction ratio causes a greater standard deviation of the error especially for moderate $T_s$. There are some remarks to be mentioned, which are consistent with our previous discussion. Increasing isolator vibration period leads to a higher mean and standard deviation error for low period superstructures. However, increasing mass ratio remarkably reduces the level of mean and dispersion of errors. The error of the proposed equations is significantly affected by the postyield stiffness ratio of superstructure where $E_{TR}$ lower than 1.2 and $\sigma_{TR}$ smaller than 0.1 are obtained for higher $\alpha_s$. Thus, the proposed equations for $C_1$ of BI structures work well with the least error for systems with heavy superstructures, high postyield stiffness ratio of superstructure, and a low-to-moderate vibration period of isolation system. Figure 20 represents mean errors for each of the independent combinations of ($\gamma_m, T_b, \alpha_s$) in the cases of FF and NF records. Similar behavior is observed in the distribution of errors for FF and NF cases. Mean errors greater than 0.9 denote that the equations can reliably predict $C_1$ in most conditions.
Some estimated $C_1$ spectra using Equation 10 are demonstrated in Figure 21 with 2 different mass ratios ($\gamma_m = 0.8, 0.95$). The scattered data and the lines represent $C_1$ spectra for various $A_p/V_p$ ratios. It can be seen that the applicability of the proposed equations is sufficiently well in the case of FF and NF records and covers the bandwidth of the scatter of actual $C_1$ data. The verification process is based on performing some extra analysis considering other values for the parameters of Table 1. For this purpose, $\alpha_s = 0.4, 0.6$ and $\gamma_m = 0.6$ are examined by using NTHA and compared with the estimation of proposed equations. Figure 22 depicts the comparative results for the selected characteristics such as $R = 2 - T_b = 2.5, 4$ seconds $- \gamma_m = 0.8$, and $\alpha_s = 0.4, 0.6$ under FF and NF records. The results are grouped into low, intermediate, and high $A_p/V_p$ ratios, and the mean $C_1$ values are presented for each. It is obvious that the results of the proposed equations fit the exact inelastic displacement ratios for the selected parameters.

To further examine the accuracy of the proposed equations, the results of exact $C_1$ values for $R = 2 - \alpha_s = 0.1 - T_b = 3$ seconds $- \gamma_m = 0.6$ are plotted together with the $C_1$ values obtained from the proposed equations in Figure 23. The results indicate that $C_1$ curves from the proposed equations could nearly cover the bandwidth of the scatter of calculated $C_1$ for FF and NF ground motions.

7 | CONCLUSIONS

This paper presents a new approach for evaluating inelastic displacement demand in BI structures by using a simplified 2DOF system. Two new simple equations were proposed to estimate the inelastic displacement ratio ($C_1$) of BI structures located in FF and NF sites. Bilinear hysteretic model was considered, and the effects of cyclic degradations and pinching were neglected as they lie beyond the scope of this study. A parametric study was conducted to examine the inelastic displacement ratios of BI structures considering the effects of isolator and superstructure system characteristics. The results indicate that different parameters of isolation and structural system affect the inelastic demand of BI structures. The influences of superstructure vibration period, $T_s$; strength reduction ratio, $R$; postyield stiffness ratio, $\alpha_s$; and mass ratio, $\gamma_m$, with constant damping ratio of 0.02, were taken into account for the superstructure. Furthermore, a simple bilinear isolator behavior was considered for FP systems assuming different vibration periods, $T_b$, and constant normalized yield force, $\mu_f = F_0$, with no damping. $A_p/V_p$ ratio of the earthquake records was chosen as a frequency content proxy. The main findings of the parametric study could be summarized as follows:

1. The increase in strength reduction ratio always leads to a rise in the inelastic displacement ratio, specifically in a short period region. The rate of increase depends on $\alpha_s$ and $A_p/V_p$ ratio.
2. The effects of postyield stiffness ratio on $C_1$ estimation was evaluated for different levels of $A_p/V_p$ ratios. The results showed that $C_1$ estimates decrease when postyield stiffness ratio rises. Furthermore, $C_1$ values are more sensitive to postyield stiffness ratio in the cases of low to intermediate $A_p/V_p$ ratios.
3. The inelastic demand of BI structures with heavy superstructures is less sensitive to the frequency content of the ground motion. It is observed that lighter superstructures could experience a lower inelastic displacement ratio in
the case of intermediate to high $A_p/V_p$ ratios. Furthermore, the dispersion values of $C_1$ are lower for heavy superstructures.

4. Higher values of $C_1$ were obtained for smaller normalized strength $Q/[(m_s + m_b)A_p]$. Therefore, $C_1$ spectra for lower $Q/[(m_s + m_b)A_p]$ ratios can be used for other levels conservatively. Moreover, the responses of BI structures with very strong isolator ($\mu/(A_p/g) > 0.3$) resemble those in FB ones.

5. Longer period of isolation system drastically increases $C_1$, specifically in low $A_p/V_p$ ratios. It was revealed that the inelastic demand of long period BI structures is more sensitive to the frequency content of the excitation.

Finally, the accuracy of estimation of the existing $C_1$ equations was investigated in the case of FB and BI structures. Afterward, we modified $C_1$ equation suggested by ASCE41-13\textsuperscript{1} to be used for BI structures. Two proposed equations were obtained based on system and excitation parameters by using a rigorous regression process. The results demonstrated a good agreement with the exact data. Further studies considering different levels of isolator strength and excitation, more complicated hysteretic models, multistory structures, soil-structure interaction, and the effects of nonstructural components are warranted as promising suggestions for further research.

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