NUMERICAL SIMULATION OF HORIZONTAL BUOYANT WALL JET*

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Abstract: This article applies the realizable \( k-\varepsilon \) model to simulate the buoyant wall jet and gives the results of cling length, centerline trajectory and temperature dilutions at certain sections. The comparison between the numerical results and Sharp’s experimental data indicates that the model is effective in estimating velocity distribution and temperature dilutions. The velocity profiles at the central plane and \( z \)-plane both show a strong similarity at certain distance from the nozzle, and the distributions of velocity and temperature dilutions also exhibit a similarity along the axial direction at centerline in the near-field. Based on the results, the article gives the corresponding relationships between the distance and the dilutions of velocity and temperature, which is useful in predicting the behavior of the wall buoyant jet.

Key words: buoyancy, wall jet, cling length, velocity profile, temperature dilution

1. Introduction
Many industrial heat wastewaters, which are discharged into the receiving water after primary treatment through pipelines, can be considered as jets. Recently, single or multi-circular jets\(^1,2\) have been frequently applied to coastal sewage disposal projects, while their dilution of the near field is less effective than that of wall buoyant jets. Because the wall buoyant jets have a larger mixing zone than circular jets, it can effectively improve the initial dilution of the water flow field. A complete theoretical system about buoyant jets has been formed\(^3,4\), while the one about wall jets is still being studied\(^5,6\). In this article, we take into account the effect of buoyancy and wall boundary on the jet. The Coanda effect\(^7\) of buoyant wall jets can increase the mixing zone between wastewater and ambient water so that the pollution in the near field can be effectively diluted. Sharp\(^8,9\) gave the relations between the clinging length and densimetric Froude number through experimental research and theoretical analysis. Combining with Sharp’s experimental data and theoretical results, this paper has comprehensively studied the flow and temperature field. The results agree well with Sharp’s experiments in clinging length, centerline trajectory and temperature dilutions. Furthermore, the present article obtains the distributions of velocity and temperature dilution and then gives the corresponding relationships between the distributions and the...
distance from the nozzle.

2. Mathematical model and calculation method
2.1 Governing equations

A wall buoyant jet with the initial velocity $U_0$, the temperature $T_0$, the density $\rho_0$, the Reynolds number $Re_0$ and the nozzle diameter $D_0$, is discharged into the static ambient water with an infinite depth, the temperature $T_a$ and the density $\rho_a$. Generally, we should use the Boussinesq approximation, which states that density difference only works on the gravitational component in the fluid with small density variation. The sketch of model and coordinate system is shown in Fig.1. The governing equations in the Cartesian coordinate system are given as follows (where $u$, $v$, $w$ are respectively the components of mean velocity in the $x$, $y$, $z$ directions):

1. **Continuity equation**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$  \hspace{1cm} (1)

2. **Momentum equations**

$$\frac{\partial (uu)}{\partial x} + \frac{\partial (vu)}{\partial y} + \frac{\partial (wu)}{\partial z} = \frac{\partial}{\partial x} \left[ 2 \nu \left( \frac{\partial u}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ \nu \left( \frac{\partial v}{\partial x} + \frac{\partial w}{\partial y} \right) \right] - \frac{1}{\rho} \frac{\partial p}{\partial x}$$  \hspace{1cm} (2)

$$\frac{\partial}{\partial y} \left( 2 \nu \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left[ \nu \left( \frac{\partial v}{\partial x} + \frac{\partial w}{\partial y} \right) \right] - \frac{1}{\rho} \frac{\partial p}{\partial y}$$  \hspace{1cm} (3)

$$\frac{\partial}{\partial z} \left( 2 \nu \frac{\partial w}{\partial z} \right) + \frac{\partial}{\partial y} \left[ \nu \left( \frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} \right) \right] + \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial z} \right) = \frac{\partial}{\partial x} \left( \nu \frac{\partial T}{\partial x} \right) + \alpha \Delta T g$$  \hspace{1cm} (4)

3. **Temperature equation**

$$\frac{\partial (uT)}{\partial x} + \frac{\partial (vT)}{\partial y} + \frac{\partial (wT)}{\partial z} = \frac{\partial}{\partial x} \left( \frac{v_t}{S_c} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{v_t}{S_c} \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{v_t}{S_c} \frac{\partial T}{\partial z} \right)$$  \hspace{1cm} (5)

4. **$k$-equation**

$$\frac{\partial (uk)}{\partial x} + \frac{\partial (vk)}{\partial y} + \frac{\partial (wk)}{\partial z} = \frac{\partial}{\partial x} \left( \nu + \frac{k}{\sigma_k} \right) \frac{\partial k}{\partial x} + \frac{\partial}{\partial y} \left( \nu + \frac{k}{\sigma_k} \right) \frac{\partial k}{\partial y} + \frac{\partial}{\partial z} \left( \nu + \frac{k}{\sigma_k} \right) \frac{\partial k}{\partial z}$$  \hspace{1cm} (6)

5. **$\varepsilon$-equation**

$$\frac{\partial (ue)}{\partial x} + \frac{\partial (ve)}{\partial y} + \frac{\partial (we)}{\partial z} = \frac{\partial}{\partial x} \left( \nu + \frac{\varepsilon}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x} + \frac{\partial}{\partial y} \left( \nu + \frac{\varepsilon}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial y} + \frac{\partial}{\partial z} \left( \nu + \frac{\varepsilon}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial z}$$  \hspace{1cm} (7)
\[
\frac{\partial}{\partial y}\left[\left(v + \frac{\nu_\epsilon}{\sigma_\epsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] + \frac{\partial}{\partial z}\left[\left(v + \frac{\nu_\epsilon}{\sigma_\epsilon}\right)\frac{\partial \varepsilon}{\partial z}\right] +
\]
\[C_1 \varepsilon \frac{\varepsilon^2}{k + \sqrt{\nu \varepsilon}} + C_3 \nu_\epsilon \frac{\partial T \varepsilon}{\partial z} = \frac{\varepsilon}{\varepsilon} \frac{\partial \varepsilon}{\partial t} \]

(7)

where \( p \) is the fluid pressure, \( g \) the acceleration of gravity, \( k \) the turbulence kinetic energy, \( \varepsilon \) the turbulence dissipation rate, \( G_k \) the generation of turbulence kinetic energy due to the mean velocity gradients and \( G_\kappa = 2\mu_k \left( E_y E_y / E \right) \), \( \mu_k \) the Prandtl number, \( \mu_\epsilon = \mu / \rho \), \( \nu_\epsilon = \nu / \rho \), \( \mu \) is dynamic viscosity of water, and \( \alpha = \frac{C_\mu}{C_\nu^2} \) the turbulence viscosity , \( C_\mu = 1/ \left[ (A_0 + A_1 U^*/\varepsilon) \right] \) the function of the mean strain and rotation rates, where \( A_0 = 4.04 \), \( A_1 = \sqrt{6}\cos \phi \), and \( \phi = \arccos(\sqrt{6}W)/3 \). \( W = E_y E_y / E \), \( E_y = \left( \partial u / \partial x \right) + \partial u / \partial y \), \( E = \sqrt{E_y E_y} \). \( v = \nu + \nu_\epsilon \), \( \nu = \mu / \rho \) is the thermal expansion coefficient of water, which changes with water temperature. Here \( \alpha \) is determined by Batchelor’s function\[16]\):

\[
\alpha = (-773 + 190T - 2.7T^2 + 0.021T^3) \times 10^{-7}
\]

where \( T \) is the temperature of local water.

2.2 Boundary conditions

This study only computes half of wall jet domain due to the symmetry of the nozzle.

(1) Inlet boundary

As \( x = 0 \), \( y^2 + (z - D/2)^2 \leq D^2 / 4 \),

\[
\begin{align*}
(y \geq 0, z \geq 0), \\
 u = U_0, \ v = w = 0, \ T = T_0, \\
k = 0.06u^2, \ \varepsilon = 0.06u^3 / D.
\end{align*}
\]

(2) Outlet boundary

Regarding the flow at outlet section as a fully developed turbulent flow leads to the second class of boundary conditions for \( u, v, w, k, \varepsilon, T \), i.e., the gradients perpendicular to the boundary are zero.

(3) On the plane of symmetry

As \( y = 0, \ v = 0 \),

\[
\frac{\partial u}{\partial y} = \frac{\partial w}{\partial y} = \frac{\partial k}{\partial y} = \frac{\partial \varepsilon}{\partial y} = \frac{\partial T}{\partial y} = 0.
\]

(4) Wall boundary

As \( z = 0, \ w = 0 \) (i.e., no-slip condition) and adiabatic wall condition are adopted. \( u, v, k, \varepsilon \) are assumed to be standard wall functions\[17\].

2.3 Computational method

The incompressible steady flow is considered in this article. Referring to the experimental data given by Sharp\[14\], we selected 25 cases and listed them in Table 1. The Finite Volume Method (FVM) combined with structured grids is applied to discretize the governing equations. And SIMPLEC scheme is used for the coupling between pressure and velocity. The computation is convergent as the residual is smaller than \( 1 \times 10^{-6} \) for the energy equation and \( 1 \times 10^{-5} \) for the other governing equations.

3. Cling length

The region near the wall can not provide enough fluid for the entrainment of the jet, so the region will have lower pressure than that on the outer side of the jet. The difference of pressure causes the jet cling to the wall and slip for some distance before it leaves the floor. As the velocity decreases, the suction pressure becomes lower and the buoyancy dominates the flow, which will leave the wall when the buoyancy forces overcome the pressure difference. Therefore, the wall buoyant jet can be divided into three regions (Fig 2): initial region, wall jet region and free jet region. The initial region is the region where the maximum velocity keeps constant, and the one behind the wall jet region is the free jet region, which behaves similarly to the free jet. According to the position of the jet lifting off, the wall jet region is further divided into Regions I and II. The region between the end of initial region and the location of the jet lifting off at central plane is named as wall jet Region I. And the other region of the wall jet region is wall jet Region II. In the wall jet Region I, the distribution of velocity and dilution exhibit similarity. For the convenience of describing the wall buoyant jets, we define the initial region and wall jet Region I as the near-field.
Table 1 Calculational conditions

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Fig. 2 Regions of flow

The cling length \( L \) (Table 1) is defined as the distance between the nozzle and the position where the temperature on the floor with the condition of \((T - T₀)/(T₀ - Tₚ) \approx 3\% \). The relation between the cling length and initial densimetric Froude number \( F_d \) is represented in the dimensionless coordinates (see Fig.3), and Sharp’s experimental results are also shown in Fig. 3. It can be seen from the figure, the experimental result and the numerical result are in good agreement. Therefore, the relationship between \( L/D \) and \( F_d \) is given as

\[
L/D = 3.2 F_d \quad (8)
\]

4. Trajectory of centerline

The centerline trajectory given by numerical computation, experiments and theoretical solutions of Sharp and Abraham at the central plane in the free jet region are shown in Fig. 4. And the computed results are close to experimental result. The comparisons indicate that the wall buoyant jet clings to the floor and slips for some distance before it lift off, and the
position that the centerline leaves wall is closer to the nozzle. The distance between the nozzle and the position where the trajectory leaves the floor is $L'$ and is determined by $L' = 2DF_d$.

Fig. 3 Length of clinging

Fig. 4 Comparison of centerline trajectories

Fig. 5 Comparison of temperature dilutions

5. Temperature dilutions

The temperature dilution can be defined as $S = (T_o - T) / (T' - T)$. which is similar to the definition for the dilution of thermal buoyant jets in co-flowing shallow water\[^{10}\]. $T_o$ is the initial jet temperature, $T_o$ is the ambient water temperature, and $T$ is the temperature on the numerical mesh node. Figure 5 shows the temperature dilutions along the centerline trajectory for several cases at the cross-section $z/D = 35$ and $z/D = 70$. The comparisons between experimental results and theoretical solutions of the dilutions can be found in Ref.\[^{12}\]. Figure 6 shows the temperature dilution profiles of Case 3 at horizontal plane of $z/D = 70$ and $z/D = 100$. The profiles appear to be pear-shaped, and the other cases also have the similar shape. The conclusion agrees well with that of Sharp’s\[^{15}\].

In view of the facts mentioned above, the clinging length, trajectory of centerline and temperature dilutions agree well with Sharp’s experimental results and theoretical solutions. Other numerical results obtained by realizable $k$-$\varepsilon$ model will be given next.

6. Characteristics of dilutions

The contours of temperature dilutions at central plane ($y = 0$) (Fig. 7) illustrates that the dilutions increase with the increasing distance and are related to the jet outlet diameter $D$ and the densimetric Froude number $F_d$. 
Temperature dilutions of some cases in the near-field change along the centerline (Fig.8), and obey the same discipline in the zone of wall jet Region 1. The article gives the best-fit curve in this region as:

$$S = 0.0725 \frac{x}{D} + 0.85$$  \hspace{1cm} (9)$$

The dilution almost keeps constant at the distance of $0 < x < 5D$, which can be defined as the boundary of the initial jet region.

7. Characteristics of velocity
7.1 Velocity profiles in central plane

Velocity profiles in the central plane in the near-field for Cases 2, 3, 10 and 12 are exhibited in Figs.9 and 10. Both figures take $u_m / u_{m0}$ as the abscissa and $z/z_{m/2}$ as the ordinate, where $u_m$ is the velocity component in the $x$ direction at $z$ of central plane, $u_{m0}$ is the maximum velocity of $u_m$ and its ordinate is $z_m$, $z_{m/2}$ is the velocity-half-height, that is, the ordinate of $u_m = u_{m0} / 2$. The velocity profiles for Cases 2 and 10 (Fig.9) exhibit similarity after $5D$, which indicates that the range of initial region is $0 < x < 5D$. Figure 10 shows the velocity distributions of Cases 3 and 12 as $x/(DF_d)$ equals the same values. As $x < 2DF_d$, the velocity profiles for different cases behave similarly to each other. Furthermore, the profiles yield the position where centerline leaves the wall is $L' = 2DF_d$. The velocity profiles exhibit a strong similarity in the range of $5D < x < 2DF_d$, and other cases also have the same shape. The empirical equation (Eq.(10)) given by Verhoff (1963) for planar turbulent wall jet\cite{18} is also plotted in the Figs.9 and 10, which both
are in good agreement and illustrate that velocity profiles are the same as that of the one for planar turbulent wall jet in the wall jet Region I.

\[
\frac{u}{u_m} = 1.48 \left( \frac{z}{z_{m/2}} \right)^{1/7} \left[ 1 - \text{erf} \left( \frac{0.68 \frac{z}{z_{m/2}}}{2} \right) \right]
\]  

(10)

7.2 Velocity distribution in z-plane

Figures 11 and 12 give the velocity profiles at the z-plane with dimensionless coordinate \( \frac{u}{u_m} \) and \( \frac{y}{b} \), where \( u \) is the velocity component at \( y \) in the plane of \( z = z_m \), and \( b \) is velocity-half-weight, that is, the ordinate of \( u = u_m / 2 \). Both the figures indicate that the velocity profiles of \( x/D \) or \( x/(DF_d) \) behave similarly when \( x > 5D \) and also show the similarity not only in the wall jet region but also in the free jet region. The article fits the distribution into the following function:

\[
\frac{u}{u_m} = \exp \left[ -\left( \frac{y}{1.2b} \right)^2 \right]
\]

(11)

7.3 Decay of centerline velocity

Figure 13 shows the decay of centerline velocity in the near-field of several cases. We use \( U_0 / u_m \) as the ordinate and \( x/(DF_d) \) as the abscissa because the length is related to the densimetric Froude number \( F_{d} \). The figure shows that the centerline velocities have a similarity, and the other cases are similar. We fit the profile to the function
\[
\frac{U_0}{u_m} = 0.65 \frac{x}{D \sqrt{F_d}} + 0.3 \tag{12}
\]

8. Conclusions
The buoyant wall jets can broaden the interface between polluted water and receiving water due to the Coanda effect and enhance the entrainment and turbulence in receiving water to form an intense dilution zone. So we can use it to rapidly improve near-field dilution for waste water discharging and to enhance the dilution effect of the sewage diffuser.

This article uses the realizable \(k-\varepsilon\) model to simulate the wall buoyant jet and compares the numerical results of the cling length, centerline trajectory and dilutions with Sharp's experimental data and theoretical results. The main conclusions are reached as follows:

1. Cling length is proportional to the outlet diameter \(D\) and initial densimetric Froude number \(F_d\) (i.e., \(L/D = 3.2F_d\)).
2. Temperature dilutions are related to the distance from outlet \(x\), outlet diameter \(D\) and densimetric Froude number \(F_d\). And the decay of temperature dilutions along with the axis in the wall jet Region I. fits to the Eq.(9).
3. Velocity profiles on central surface are similar to that of planar turbulent wall jet, and are in good agreement with the classical wall jet curve.
4. Velocity profile exhibits excellent similarity in the \(z\)-plane after certain distance \((x > 5D)\), and fits to Gaussian shape. Function is the Eq.(11).
5. The decay of centerline velocity is related to the outlet diameter \(D\) and densimetric Froude number \(F_d\), and fits to the Eq.(12).

References