EXPLORING FERROFLUIDS FOR HEAT TRANSFER AUGMENTATION

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ABSTRACT
We explore the potential of ferrofluids for heat transfer augmentation, via numerical simulations using water-based ferrofluids flowing between parallel plates (laminar flows; Reynolds number (Re) < 100), exposed to external magnetic fields, produced by single and multiple magnetic line dipoles. Two water-based ferrofluids having different magnetic and thermo-physical properties owing to their different solid volume fractions of magnetic nanoparticles are considered. It is shown that the presence of the non-uniform magnetic field produced by the dipoles induces an attractive magnetic body force on ferrofluids, which disturbs the flow locally, thus augmenting local Nusselt number. While flow strongly gets affected by the magnetic field at Re = 25, magnetic field does not affect the flow significantly beyond Re ≥ 75 which shows the dominance of inertial forces over magnetic forces, at a given external magnetic field strength. A detailed parametric study is performed to investigate the effect of single and multiple dipole placements, the solid volume fraction of magnetic nanoparticles and Reynolds number on heat transfer enhancement. It is concluded that forced convective heat transfer can be enhanced by applying external magnetic field compared to no magnetic field case, only under certain boundary conditions.

Keywords: Ferrofluids; Magnetic field; Convective heat transfer; CFD simulations; Transport phenomena
## NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$M$</td>
<td>Magnetization [A/m]</td>
</tr>
<tr>
<td>$H$</td>
<td>Magnetic field [A/m]</td>
</tr>
<tr>
<td>$B = \mu_0(H+M)$</td>
<td>Magnetic flux density or magnetic induction [T]</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature [K]</td>
</tr>
<tr>
<td>$u, v$</td>
<td>Velocity components in $x$ and $y$ directions [m/s]</td>
</tr>
<tr>
<td>$v_{avg} = \sqrt{u^2 + v^2}$</td>
<td>Average velocity [m/s]</td>
</tr>
<tr>
<td>$x, y$</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>$r, \phi$</td>
<td>Polar coordinates</td>
</tr>
<tr>
<td>$X, Y$</td>
<td>Coordinate of magnetic dipoles</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant [J/K]</td>
</tr>
<tr>
<td>$L$</td>
<td>Channel length [m]</td>
</tr>
<tr>
<td>$h$</td>
<td>Plate separation [m]</td>
</tr>
<tr>
<td>$D$ or $D_h$</td>
<td>Hydraulic diameter (2$h$) [m]</td>
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<tr>
<td>$m$</td>
<td>Magnetic dipole strength [A-m]</td>
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<tr>
<td>$d$</td>
<td>Nanoparticle diameter [m]</td>
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<tr>
<td>$f_I$</td>
<td>Magnetic body force [N/m$^3$]</td>
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<td>$q''$</td>
<td>Heat flux [W/m$^2$]</td>
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<td>$k$</td>
<td>Thermal conductivity [W/m-K]</td>
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<tr>
<td>$C$</td>
<td>Specific heat [J/kg-K]</td>
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<td>---------------------------</td>
<td>-----------------------------------------------------------------</td>
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<tr>
<td>$\varphi$</td>
<td>Solid volume fraction [-]</td>
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<td>$\alpha$</td>
<td>Langevin parameter [-]</td>
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<td>$\mu_0$</td>
<td>Permeability of free space [H/m]</td>
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<tr>
<td>$\mu$</td>
<td>Dynamics viscosity [Pa-s]</td>
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<tr>
<td>$\rho$</td>
<td>Density [kg/m$^3$]</td>
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<table>
<thead>
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<th>Subscripts</th>
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<tr>
<td>$bf$</td>
<td>Base fluid used in ferrofluid preparation</td>
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<tr>
<td>$ff$</td>
<td>Ferrofluid</td>
</tr>
<tr>
<td>$s$</td>
<td>Solid</td>
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<tr>
<td>$0$</td>
<td>Ambient or reference</td>
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<tr>
<td>$w$</td>
<td>Wall</td>
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<td>$b$</td>
<td>Bulk</td>
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<table>
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<th>Non-dimensional numbers</th>
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<tr>
<td>Reynolds number (Re)</td>
<td>$\frac{\rho u D_h}{\mu}$</td>
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<tr>
<td>Prandtl number (Pr)</td>
<td>$\frac{\mu C}{k}$</td>
</tr>
<tr>
<td>Nusselt number (Nu)</td>
<td>$\frac{h D_h}{k}$</td>
</tr>
<tr>
<td>Magnetic Froude number (Fr$_m$)</td>
<td>$\frac{\rho v_{avg}^2}{\mu_i M_s L(\alpha) H}$</td>
</tr>
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</table>
INTRODUCTION

Low thermal conductivity of traditional heat transfer fluids, such as water or oil, has been a major bottleneck in increasing the thermal performance of heat exchange equipment. Nano-fluids were introduced sometime back to potentially circumvent this issue [1-2]. Such fluids are synthesized by creating a uniform and stable suspension of nanoparticles of solid materials (such as metallic oxides of copper, aluminium, zinc, or non-metallic materials, such as carbon etc.) of the size range under 100 nm in the conventional heat transfer fluids. By doing this, the thermo-physical properties of the base fluid gets altered, for example, the thermal conductivity of the nanofluid can be considerably higher than the base fluid [3-4].

Investigation of thermo-physical and transport behaviour of nanofluids under different operating conditions is receiving intense contemporary focus. Heat transfer augmentation studies with nanofluids is being investigated in a wide range - from natural and forced convection [5-8], active and passive two-phase heat transfer [9, 10] to heat exchangers [11] and in porous media [12, 13]. Ferrofluids are a special class of nanofluids, which utilize magnetic materials as the dispersed phase, hence having an additional advantage of possessing magnetic properties as well. These magnetic properties can be used to further manipulate the flow thereby strongly influencing the ensuing heat transfer and other transport characteristics of ferrofluids [14, 15].

Magnetic nanofluids or ferrofluids are a stable colloidal suspension of single domain magnetic nanoparticles (e.g. magnetite, maghemite) in a conventional fluid such as water or an organic solvent. For the stability of the colloidal solution, magnetic nanoparticles are surface treated to induce steric/ionic repulsion among nanoparticles. This surface treatment avoids aggregation and settling of nanoparticles due to Van der Waals forces and dipole-dipole interactions [16]. Nanoparticles are usually coated with long-chain polymers (e.g. Oleic acid (C₁₈H₃₄O₂), Citric acid (C₆H₈O₆), Lauric acid (C₁₂H₂₄O₂), Tetramethylammonium hydroxide (N(CH₃)₄OH)) to fulfil this requirement [15, 17]. Apart from this, particle sizes should also be small (2-10 nm) for thermal fluctuations to be dominant.

Magnetic nanoparticles exhibit superparamagnetic characteristics due to their single domain magnetization at room temperature. In the absence of an external magnetic field, nanoparticles are in random orientations, and the net magnetization of the ferrofluids is zero. When ferrofluids are exposed to an external magnetic field, magnetic nanoparticles start to align along the direction of the external field, giving rise to non-zero net fluid magnetization.
A body force gets induced in ferrofluids in non-uniform magnetic fields due to this net fluid magnetization. This induced magnetic body force can be used to influence/manipulate the flow characteristics of the ferrofluids [14].

Ferrofluids were invented in the early 1960s for the space applications by NASA [18]. Earlier investigations on synthesis and applications of ferrofluids were performed by Rosensweig, Raj and Moskowitz [14]. Subsequently, many new applications have been explored, such as, magnetic rotary seals, cooling of high-end speakers and power transformers, which are indeed well established industrial applications now [19]. In the last couple of decades, many new applications of ferrofluids and magnetic nanoparticles based creative technologies have also emerged. Some of the examples include energy harvesting [20, 21], sensing and actuation [22, 23], micro magneto-fluidics [24], lab-on-chip devices [25], and special applications in biomedical sciences (such as, sorting of biomolecules, magnetic drug targeting, enhancement of magnetic resonance image contrast and magnetic hyperthermia due to the biocompatibility of ferrofluids, to name a few) [15, 26, 27].

Numerical and experimental studies performed by various investigators suggest that ferrofluids can significantly enhance heat transfer, compared to pure fluids and other nanofluids, by flow manipulation using external magnetic fields. Comprehensive reviews on heat transfer enhancement using ferrofluids in several applications can be found in review articles by Nkurikiyimfura et al. [28] and Bahiraei et al. [29]. The effect of the external magnetic field on thermo-physical properties of ferrofluids and relative enhancement in thermodynamic convection is covered in the review work by Alsaady et al. [30]. An experimental investigation conducted by Philip et al. [31], to study the effect of magnetic field on thermal conductivity of ferrofluids observed ~300% increment in thermal conductivity under the external magnetic field. They observed the formation of the chain-like structures of magnetic nanoparticles in ferrofluids when exposed to external magnetic fields thus giving rise to net effective thermal conductivity. Azizian et al. [32] experimentally studied the effect of magnetic field on the laminar forced convective heat transfer of ferrofluid flowing through a stainless steel (SS) tube of inner diameter of 5.54 mm, and a length of 1 m in the presence of magnetic field produced by many pairs of permanent magnets positioned along the length of the tube. They observed a 40% increase in convective heat transfer for magnetic field gradients ranging between 8.6 - 32.5 mT/mm. Ganguly et al. [33] numerically investigated heat transfer augmentation by ferrofluids under the non-
uniform magnetic fields produced by single and multiple magnetic dipoles of different strength at Reynolds number of 11. They reported a relatively large local change in heat transfer coefficient at dipole locations due to a large change in local velocities associated with induced magnetic body forces by the dipoles. Heat transfer enhancement observed to be strongly dependent on the strength of magnetic dipoles in their study. Goharkhah et al. [34] investigated the effect of the alternating non-uniform magnetic field on fluid flow and heat transfer using the similar mathematical model as Ganguly, in parallel plate configuration. They reported further improvement in heat transfer coefficient, which was dependent on the frequency of the alternating magnetic field. A maximum of 13.9% enhancement was observed in their study. Steak et al. [35] performed a numerical study using a similar formulation. They observed that the flow was relatively uninfluenced by the magnetic field until its strength was large enough for the Kelvin body force to overcome the inertial forces.

In most of the investigations, authors have considered the linear dependence of ferrofluid magnetization on the applied magnetic field through magnetic susceptibility (χ) of ferrofluids, which is only valid for a narrow range of applied magnetic fields. As magnetization of the ferrofluids reaches saturation in strong applied magnetic fields, this linear model does not provide accurate numerical predictions due to associated nonlinearity in the fluid magnetization. The nonlinear behaviour of ferrofluids in external magnetic and temperature fields also give rise to several associated phenomena, such as normal field instability (also known as Rosensweig instability) [36], Labyrinthine instability [37], Kelvin-Helmholtz instability and Rayleigh-Marangoni-Bénard instability [38, 39]. These phenomena have also been observed experimentally and studied extensively, both for their intriguing physical nature, and practical relevance [14,15]. Recent work by Esmailzadeh et al. [40] on analytical methods for different nonlinear problems is useful in analytically studying the nonlinear behaviour of ferrofluids.

In most of the earlier studies, nonlinearities in ferrofluid magnetization have either been simplified or neglected. Although few studies have implemented this inherent nonlinearity associated with fluid magnetization, literature is largely scarce in this area. Banerjee et al. [41] have implemented the Langevin magnetization model for their numerical study; however, their work is focused on natural convection in ferrofluids. Sheikhnjejad et al. [42] have implemented the Langevin model for their study on forced convection laminar heat transfer in tube configuration, but their work is on the axial differential magnetic field, which is not very common in practice. Special setups are required to produce such magnetic fields
in real life experiments. An experimental study on forced convective heat transfer in parallel plates configuration has been performed by Goharkhak et al. [43] utilizing numerical simulations to choose the most effective configuration of magnets to maximize heat transfer enhancement for their experiments. In a recent study, an experimental investigation of laminar forced convection with ferrofluid in the external magnetic fields by Asfer et al. [44] showed significant improvement in heat transfer.

It is evident from the literature review that there are relatively fewer studies which have considered nonlinear characteristics of ferrofluids magnetization, even though it is inherent nature of the ferrofluids. In most of these studies, the linear magnetization of ferrofluid is considered, which is only valid for weak magnetic fields [39]. This linear magnetization is modelled through magnetic susceptibility ($\chi$) of ferrofluids and magnetization as well as magnetic forces is derived in terms of fluid susceptibility. The experimental measurement of ferrofluid magnetization shows additional characteristics of saturation and nonlinear regions.

In the present work, laminar forced convective heat transfer is studied in a parallel plate arrangement under external magnetic fields produced by single and multiple magnetic line dipoles which are placed along the flow direction. These dipoles produce static non-uniform magnetic field distribution which is easily deployable for real-life applications. Similar magnetic field distributions can be produced by employing permanent magnets or electromagnets. The objective of the presented study is to provide an accurate numerical model which can be utilized to assist in designing real-life experiments, for the investigation of heat transfer enhancement using ferrofluids.

PROBLEM DESCRIPTION

Steady-state laminar forced convection of water-based ferrofluid is modelled as homogenous single-phase flow in parallel plate channel of 50 mm length and 2 mm plate separation under uniform heat flux condition. Figure 1 (a) shows schematic of the problem statement with one, two and three dipole cases. Two dimensional (2D) static non-uniform localized magnetic fields are produced by simulated magnetic line dipoles along the channel, following the Maxwell electromagnetic laws [33]. Effect of single and multiple magnetic dipoles ($m$), of magnetic strength 0.5 A-m each, placed at 1 mm separation from the plates along the flow direction, is investigated for Reynolds numbers (Re) < 100 cases. Uniform heat flux boundary condition is applied to both the plate walls. Hydro-dynamically fully developed flow profile is defined at the inlet, and the flow is considered to be thermally developing.
Three different cases of dipole placements are studied, as follows

(i) Single dipole placed at 25 mm from the inlet, at 1 mm below the lower plate (figure 1a (i)).

(ii) Two dipoles placed at 1 mm below the lower plate, at 20 mm and 30 mm from inlet respectively (figure 1a (ii)).

(iii) Three dipoles placed on both side of the parallel plates at 1 mm separation from the plates at 15, 25 and 35 mm from the inlet, respectively. Dipole at $x = 25$ is placed at 1 mm above the upper plate, and the other two are placed 1 mm below the lower plate (figure 1a (iii)).

**MATHEMATICAL MODELING**

The magnetization of ferrofluids considered in the study is modelled according to the Langevin magnetization theory, as applicable to super-paramagnetic materials. Single domain magnetic particles exhibit superparamagnetic behaviour. Langevin magnetization model is based on the Boltzmann statistical average of the net magnetic dipole moment of spherical nanoparticles aligned along the direction of the externally applied field at equilibrium. The net alignment of magnetic moments is calculated through a balance between magnetic energy to thermal energy of the nanoparticles [14]. Figure 1 (b) shows the Langevin curves for the studied ferrofluids at room temperature plotted with the help of available catalogued data [45]. It is evident from this figure that magnetization curves have both nonlinear and saturation zones. Ferrofluid #1 (EMG-308, $\phi = 1.2\%$) shows saturation magnetization of $0.5 \times 10^4$ (A/m) at external field strength of $1 \times 10^5$ (A/m) and above. Similarly, Ferrofluid #2 (EMG-805, $\phi = 3.6\%$) shows saturation magnetization of $1.5 \times 10^4$ (A/m) at external field strength of $1 \times 10^5$ (A/m) and above. The localized non-uniform magnetic field is produced along the parallel plate channel by simulating magnetic dipoles which obeys Maxwell laws of electromagnetism. The magnetic field of the dipoles is simulated according to equation 3 and 4 with appropriate boundary conditions to produce the 2D non-uniform magnetic field. Langevin model takes into account the magnetic saturation of nanoparticles at strong magnetic fields. It is assumed that particles are monodispersed, dipole-dipole interactions are not present between nanoparticles, and there are no agglomerations of particles which are the characteristic of a stable colloidal solution. Hydro-dynamically fully developed and thermally developing laminar flow of ferrofluid is modelled in a parallel plate channel configuration.
Magnetic forces are modelled as body forces being exerted on the laminar flow. A uniform heat flux of $10^4$ W/m$^2$ is applied on both sides of the channel.

Magnetic field distribution due to magnetic line dipole is as per the Gauss’s and Ampère’s law (Maxwell equations), which is written as [33, 34],

$$\nabla \cdot \vec{B} = 0; \nabla \times \vec{H} = 0$$

(1)

The base fluid is non-conductive (water). Hence, the induction current in the fluid medium can be neglected.

Magnetic induction in ferrofluid domain is written as [33, 34],

$$\vec{B} = \mu_0 \left( \vec{H} + \vec{M} \right)$$

(2)

Magnetic scalar potential ($V_m$) of a magnetic line dipole can be written as [33],

$$V_m = \frac{m \sin \phi}{r}$$

(3)

Corresponding External magnetic field ($\vec{H}$) is written as,

$$\vec{H}(r, \phi) = \frac{m}{r^2} (\sin \phi \hat{r} - \cos \phi \hat{\phi})$$

(4)

Here, $m$ represents the strength of the magnetic dipole. $r$ and $\phi$ are the polar coordinates defined in the Cartesian coordinates as,

$$r = \sqrt{(x-X)^2 + (y-Y)^2}$$

(5)

$$\phi = \tan^{-1} \left( \frac{y-Y}{x-X} \right)$$

(6)

Where, $(X, Y)$ is the coordinate location of the dipole from the origin as shown in the schematic of figure 1(a). Magnetization ($M$) of ferrofluids is modelled according to Langevin function as,
\[ \vec{M} = M_s L(\alpha) \frac{\vec{H}}{|\vec{H}|} \]  

(7)

\( M_s \) is the saturation magnetization of the ferrofluids, which is also the maximum achievable magnetization in a ferrofluid sample. \( L(\alpha) \) in the above equation is called the Langevin parameter and defined as [14],

\[ L(\alpha) = \coth(\alpha) - \left( \frac{1}{\alpha} \right) ; \quad \alpha = \frac{\pi \mu_s M_s H d^3}{6 k_B T} = \frac{mH}{k_B T} \]  

(8)

where \( \alpha \) is the ratio of magnetic energy to thermal energy.

Magnetization (\( M \)) of the ferrofluids depends on particle size of the solid magnetic material chosen as the dispersed phase (\( d \)), Temperature of the fluid (\( T \)) and the applied external magnetic field (\( H \)). The saturation magnetization of ferrofluids depends on solid volume fraction (\( \varphi \)) and domain magnetization (\( M_d \)) of bulk magnetic material used for the synthesis of nanoparticles, as per the following equation,

\[ M_s = \varphi M_d \]  

(9)

The magnetic body force (\( f_k \)) on ferrofluids is calculated according to the following relation [14]. It is equivalent to the force applied by one dipole on another dipole,

\[ \vec{f}_k = \mu_0 (\vec{M} \nabla) \vec{H} \]  

(10)

After substitution of \( M \) from equation 8 and further simplification, equation 10 can be rewritten in the following form,

\[ \vec{f}_k = M_s L(\alpha) \nabla \vec{H} \]  

(11)

It is clear from the above equation that magnetic body force on ferrofluids is dependent on the gradients of the applied magnetic field and the ferrofluid magnetization. It is also clear from equation 11 that spatially uniform magnetic fields cannot induce any magnetic force, and thus will not disturb the flow.

The governing equations for Mass, Momentum, and Energy balance in 2D geometry for the incompressible flow, neglecting viscous dissipation, is written as
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (12)

\begin{align}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + f_k(x) \quad (13-a)
\frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + f_k(y) \quad (13-b)
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{k}{\rho} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (14)
\end{align}

The boundary conditions for the above set of equations are as under:

At flow inlet \((x = 0)\), fully developed velocity profile is defined,
\[ u = 3 \frac{v_{\text{avg}}}{2} \left(1 - 4 \left(\frac{y}{2}\right)^2\right); \quad v = 0; \quad T = T_{\text{in}} \quad (15-a)\]

At fluid-wall interface \((0 \leq x \leq L, \; y = 0 \text{ or } y = h)\), no slip and uniform heat flux boundary conditions are applied,
\[ u = v = 0; \quad q^*_{\text{w}} = -k \frac{dT}{dy} \quad (15-b)\]

At exit \((x = L)\), outflow boundary condition is defined as,
\[ p = p_0; \quad \frac{dT}{dx} = 0 \quad (15-c)\]

Equations 1 to 11 are solved to calculate the magnetic force term \((f_k)\) which is then implemented in \(x\) and \(y\) momentum equations 13-a and 13-b as external body forces. Gravity is neglected as the gap between plates is small. As fluid magnetization is dependent on fluid temperature, equations 1 to 14, with boundary conditions (equations 15-a-c) are solved simultaneously as closed coupled equations to get the solution. Local and average Nusselt numbers are calculated using following equations.
\[ Nu = \frac{q^*h}{(T_c - T_h)}; T_h = \left[ \int_0^h \langle UT \rangle dy \right] / \left[ \int_0^h U dy \right] \] (16)

\[ \overline{Nu} = \left( \frac{1}{L^2} \right) \int_0^L Nu \, dx \] (17)

where \( T_h \) is the bulk fluid temperature.

The effect of inertia force over magnetic force is quantified through a dimensionless number. This non-dimensional number is termed as magnetic Froude number in this study. In general, Froude number is defined as the ratio of inertia force to the force due to an external field. The magnetic Froude number is scaled with the characteristic hydraulic diameter \( (D_h) \), and is defined as follows,

\[ Fr_m = \frac{\rho v_{avg}^2}{\mu_0 M_1 M_2 (\alpha) H} \] (18)

More details on the scaling of different forces used in ferrohydrodynamics can be found in the review article of Nguyen [24].

**Magnetic and thermo-physical properties**

Magnetic and thermo-physical properties of the ferrofluid are taken from the catalogue data provided by Ferrotec®, the supplier of the material [45]. Other required properties, such as thermal conductivity and specific heat, which are unavailable in the catalogue, are estimated using widely accepted standard correlations used in the literature on ferrofluids [34, 43, 44, 46]. Thermal conductivity is estimated using classical Maxwell correlation [47, 48] given for dilute monodispersed suspension of spherical nanoparticles (equation 19) and specific heat is estimated according to correlation shown in equation 20 [49]. Table 1 and Table 2 contain thermo-physical and magnetic properties of both the ferrofluids which are assumed to be constant over the studied temperature range.

\[ k_{ff} = \frac{k_s + 2k_{bf} - 2(k_{bf} - k_s) \phi}{k_s + 2k_{bf} + (k_{bf} - k_s) \phi} k_{bf} \] (19)
\[ c_{ff} = \frac{\rho_s c_s \phi + \rho_{ff} c_{ff} (1-\phi)}{\rho_{ff}} \]  

(20)

**Table 1:** Thermo-physical properties of ferrofluids

<table>
<thead>
<tr>
<th>Ferro-Fluid</th>
<th>Density ((\rho_{ff}, \text{kg/m}^3))</th>
<th>Specific heat ((C_{ff}, \text{J/kg.K}))</th>
<th>Thermal conductivity ((K_{ff}, \text{W/m.K}))</th>
<th>Viscosity ((\mu_{ff}, \text{Pa-s}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMG-308</td>
<td>1060</td>
<td>3915.9</td>
<td>0.63</td>
<td>0.002</td>
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<tr>
<td>EMG-805</td>
<td>1190</td>
<td>3475.2</td>
<td>0.67</td>
<td>0.003</td>
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</tbody>
</table>

**Table 2:** Magnetic properties of ferrofluids

<table>
<thead>
<tr>
<th>Ferro-Fluid</th>
<th>Solid volume fraction ((\phi, %))</th>
<th>Saturation Magnetization ((M_s, \text{mT}))</th>
<th>Magnetic susceptibility ((\chi_{ff}, \text{-}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMG-308</td>
<td>1.2</td>
<td>6.6</td>
<td>0.50</td>
</tr>
<tr>
<td>EMG-805</td>
<td>3.6</td>
<td>22</td>
<td>2.89</td>
</tr>
</tbody>
</table>

**NUMERICAL METHOD**

Due to the non-linear nature of the coupled equations, a numerical approach is taken to solve these equations. Finite element formulation (FEM) based commercial software Comsol Multiphysics® has been used to simultaneously solve strongly coupled Multiphysics equations of mass, momentum and energy balance and the Maxwell equations for incorporating the magnetic field physics. On this platform, a fully coupled approach employing damped Newton-Raphson method is used. Adaptive mesh refinement feature is incorporated to accurately resolve the force and field gradients induced by the magnetic field in fluid flow. This feature calculates the gradient of the variables in the first step; subsequently, more mesh elements are added in the regions of large gradients, and all equations are solved again on the newly generated mesh structure in the next solution steps. The Parallel Sparse Direct Solver (PARDISO), which is based on LU decomposition, is employed to solve the system of equations formed at every Newton-Raphson iteration steps during computation. The purpose of the numerical simulation is to solve for Magnetization \((M)\) in ferrofluids, Magnetic body force \((f_s)\), Flow velocities \((u, v)\) and Temperature \((T)\) fields in the computational domain.

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Validation of Numerical Models

A validation study is performed to establish the accuracy of obtained numerical solutions. For this purpose, a separate mathematical model was implemented in Comsol® according to equations provided in the article by Ganguly et al. [33]. Schematic of their problem is shown in figure 2 (a). Hot ferrofluid enters at 380 K from the inlet and being cooled by lower plate maintained at 300 K, and the upper plate is maintained in the adiabatic condition. Flow is disturbed by the magnetic body force induced in ferrofluid by external magnetic field. The static non-uniform magnetic field is produced by the simulated line dipoles along the channel. Their mathematical model for ferrofluid magnetization and magnetic body force is shown in equations 21 to 23 below.

Magnetic induction in ferrofluid medium is modelled as following

\[ B = \mu_0(1 + \chi_m) m \left[ \frac{\sin \varphi}{r^2} \hat{r} - \frac{\cos \varphi}{r^2} \hat{\varphi} \right] \]  

(21)

The magnetization of ferrofluids is modelled as the linear dependence of magnetization on the external magnetic field \((H)\) through magnetic susceptibility \((\chi_m)\) of ferrofluids, which is again defined as a temperature dependent property as followed

\[ \frac{M}{H} = \chi_m = \chi_m(T) = \chi_0/[1 + \beta(T - T_0)] \]  

(22)

The magnetic body force is derived from the general form of equation (equation 10) as followed

\[ f_k = \frac{1}{2} \mu_0 \chi_m (1 + \chi_m) \nabla (H.H) + \mu_0 \chi_m H (H.\nabla \chi_m) \]  

(23)

Equations 21 to 23, coupled with fluid flow and heat transfer equations (equation 12 to 15) were solved simultaneously on Comsol® Multiphysics. Results of the Comsol simulations are then compared with results of Ganguly et al. [33]. Quantitatively as well as qualitatively similar results for velocity and temperature fields are obtained, which provides the necessary confidence in the simulation framework. The difference between the two results is because of different numerical schemes used to solve coupled mass, momentum, energy and magnetic field equations in both the studies. While Ganguly used finite difference based SOLA code developed by Hirt et al. [50], finite element based commercial code Comsol® Multiphysics with multiple steps of adaptive mesh refinement is used in the present study to resolve magnetic field and force gradients accurately.
In figure 2 (b), the local Nusselt number is plotted against the results of Ganguly at dipole strength of 0.58 Am showing similar comparative trend according to the present computations. Results for Nusselt number trends for no magnetic field cases are also compared with the analytical solution for forced convective flow under uniform heat flux condition as given in Shah and London [51] for thermally developing flow in the parallel plate configuration and observed to be matching accurately, as seen in figure 2 (c). Analytical expression of Shah and London used for validation is shown below (equation 24).

\[
Nu_x = \begin{cases} 
1.490(x^*)^{-0.5} & \text{if } x^* < 0.0002 \\
1.490(x^*)^{-0.5} - 0.4 & 0.0002 \leq x^* < 0.001 \\
8.235 + 8.68(10^3x^*)^{0.508}e^{-164x^*} & x^* \geq 0.001 
\end{cases}
\]

where, \( x^* = \frac{x}{D_0} \frac{Re Pr}{\mu} \)

**RESULTS AND DISCUSSION**

Hydro-dynamically fully developed flow of water-based ferrofluid flows into the parallel plate channel at 298 K and is heated by uniform heat flux applied to both the plates. While passing through the channel, flow is affected by the static non-uniform magnetic field produced by the magnetic line dipoles placed along the channel. Figure 3 (i) and (ii) show the contour plots of the static spatially non-uniform magnetic field produced by the magnetic dipoles and induced magnetic body force \( f_k \) in ferrofluids for all cases of dipole placement. It is evident from the plots that the magnetic field and its gradients are high in the close vicinity (up to 5 mm on both sides) of dipole locations. The maximum strength of the magnetic field is \( \sim 5 \times 10^5 \) A/m right above the position of the dipoles which is well above the magnetic strength required to achieve saturation magnetization in the ferrofluid. The maximum strength of magnetic body force \( f_k \) is \( \sim 6.48 \times 10^8 \) N/m³ for EMG-308 (\( \varphi = 1.2\% \)) and \( \sim 2.21 \times 10^7 \) N/m³ for EMG-805 (\( \varphi = 3.6\% \)) ferrofluid. In figure 4 (i) and (ii), Magnetic field and forces are plotted along the central line of the channel for all cases of dipole placement. It clearly shows that the field distribution of a single dipole is symmetric and effective up to 5 mm on either side of the dipole location. The separation between two dipoles is kept at 10 mm to study the effect of each dipole individually. Three different Reynolds number (Re) of 25, 50 and 75 are investigated to study the effect of inertial/viscous forces over magnetic forces.
Figures 5 to 7 show the steady-state spatial variations of temperature (contour plots) and velocity (vector plots), with and without magnetic fields, for both the ferrofluids. It is apparent from the plots that the presence of dipoles creates a localized effect on the velocity field creating a recirculation zone in the fluid flow near dipole locations. All such recirculation zones are visible in figures which are induced near the dipole locations due to magnetic body forces. The recirculation of fluid creates stagnation and acceleration zone in fluid flow in the close vicinity of the dipole locations and enhances localized mixing of fluid by disturbing thermal boundary layers. The disturbance in thermal boundary layers can also be seen in figures 5 to 7 for all three studied cases. In figure 2 (d), the velocity profile along the y-axis is plotted for the cases of the magnetic field due to the single dipole and no magnetic field right above the dipole location. It clearly shows that the flow velocity is relatively quite small near the lower plate and accelerates in between due to the magnetic field effect. This effect is narrow at Re = 75 but extends to almost one-third of the plate separation for Re = 25. The recirculation of fluid also gets smaller as Reynolds number is increased, as seen in figure 5 (c) and (d) for Re = 50 and Re = 75, respectively. This shows that a direct relationship exists between inertia and magnetic forces. The inverse of magnetic Froude number ($Fr_m^{-1}$) is calculated to quantify this relationship and compare the effect of magnetic force over inertia force for different Reynolds number. The magnetic field ($H$) and fluid velocity ($v_{avg}$) is non-uniform in the y-direction, so an average of these quantities is calculated through line integral across the channel in the y-direction, right above the dipole locations. Afterwards, the inverse of the magnetic Froude number is calculated using these average values. The inverse of magnetic Froude number ($Fr_m^{-1}$) for EMG-805 ferrofluid is $\sim 4.2 \times 10^3$ at Re = 25, $\sim 1.4 \times 10^3$ at Re = 50 and $\sim 7.7 \times 10^2$ at Re = 75. Because the magnetic field is static, the magnetic force and denominator of the magnetic Froude number representing magnetic force term is constant. As the Reynolds number is increased, the inverse of magnetic Froude number ($Fr_m^{-1}$) changes thrice for Re = 50 to six times for Re = 75, from its value at Re = 25, which means that the strength of inertia force has increased six times from its strength at Re = 25. Beyond Re = 75, no significant change in observed in local and average Nusselt number compared to no magnetic field cases indicating that the effect of the magnetic field has become insignificant.

The stagnation/acceleration of flow near lower wall causes a slight dip and steep rise in local Nusselt number (Nu) on the lower plate as seen in plots of figures 8 (a) and 9 for the bottom plate Nusselt number. However, this localized mixing of the fluid increases the bulk fluid
temperature as seen in figures 8 (b) and 10, causing the overall average Nusselt number to rise. The local Nusselt number at the upper plate is always high near dipole locations and goes as high as 20, as seen in plots of figures 8 (a) and 9 (b), (d) compared to no magnetic field cases due to the acceleration of flow near the upper plate, which is almost 250% higher than the local Nusselt number at that point for no field cases. This acceleration of fluid can also be seen in the velocity vector plots of figures 5 to 7 where longer vectors represent higher magnitude. For three dipoles placement case on both side of the plates, flow is affected on both side of the channel, as seen in figure 9 (e) and (f) and better mixing of fluid is observed.

The strength of the magnetic force is more for higher solid volume fraction, resulting in a relatively larger recirculation of ferrofluid, which can also be seen in the presented figures. The effect of two and three dipoles placement on velocity and temperature is shown in the figures 6 and 7 for both the fluids. The presence of multiple dipoles disturbs the flow at multiple locations which adds to corresponding heat transfer enhancement. Similar observations for flow behaviour in the presence of dipoles have also been reported by other authors [33-35] in their respective studies.

Average Nusselt numbers ($\overline{Nu}$) for all the studied cases of the two ferrofluids is shown in Table 3 and plotted against their Reynolds number in figure 11. A comparison of data from Table 3 and both the plots in figures 11 shows that higher solid particle loading gives better results for all cases of magnetic fields and Reynolds number, which is expected, given that higher solid content results in superior thermo-physical properties of ferrofluids. At Re = 25, both the ferrofluids show best results for three dipoles placement, when compared to the rest of the cases which shows that the placement of multiple magnets along the plate aids in heat transfer enhancement at low Re. Maximum average Nusselt number ($\overline{Nu}$) of 11 for EMG-308 and 12 for EMG-805 is achieved at Re = 25, for three dipoles placement case, as seen in figure 11, which is 22% and 28% higher compared to no magnetic field case, respectively. As Reynolds number is increased to 50 and 75, the average Nusselt number rises for the no field, and single dipole cases, and a slight dip and rise is observed for the two and three magnet cases at Re = 50 and Re = 75, respectively. It is because of the balance between magnetic and inertial forces at Re = 50 and dominance of later at Re = 75. For Re ≥ 75 cases, the average Nusselt numbers for single and multiple magnets cases are almost identical, as in no magnetic field cases for both the fluids which shows that the effect of the magnetic field is insignificant at the applied field strength (m = 0.5 Am) and inertial forces are dominant.
Table 3: Average Nusselt number for all studied cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Average Nusselt number (( Nu ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EMG-308</td>
</tr>
<tr>
<td></td>
<td>EMG-805</td>
</tr>
<tr>
<td>Flow Reynolds number (Re)</td>
<td>25</td>
</tr>
<tr>
<td>No magnet</td>
<td>9.02</td>
</tr>
<tr>
<td>Single magnet</td>
<td>9.82</td>
</tr>
<tr>
<td>Two magnets</td>
<td>10.36</td>
</tr>
<tr>
<td>Three magnets</td>
<td>10.98</td>
</tr>
</tbody>
</table>

The above presented mathematical modelling and numerical analysis can be employed to accurately determine the strength of magnetic forces at any particular location of interest in the ferrofluid domain and the range over which magnetic forces are effective for a given strength of the magnetic dipole. By incorporating both nonlinear and saturation characteristics of the ferrofluid magnetization, we have overcome the limitations of previously presented linear models. Our model can also be used to study the nonlinear behaviour and associated phenomena displayed by ferrofluids in external magnetic and temperature fields, as stated in the introduction section. The presented model can be applied to study thermo-magnetic convection in both 2D and 3D geometries. We have considered the magnetic field produced by line dipoles in our study. A realistic magnetic field produced by a magnetic dipole of finite size can be obtained by arranging a collection of such line dipoles [33]. It can be used designing real-life experiments to study heat transfer enhancement due to thermomagnetic convection, an example of which can be found in Goharkhah et al. for the case of heat transfer in parallel plates [43]. Further, the model can be effectively used in other areas involving ferrofluid flow in external magnetic fields such as micro-magnetofluidics, Lab-on-Chip devices and in biomedical applications (some examples include magnetic drug targeting and sorting of biomolecules using magnetic fields).
SUMMARY AND CONCLUSION

Laminar forced convection of ferrofluids is studied in a parallel plate setup under external non-uniform magnetic field produced by magnetic line dipoles placed along the length of the channel. Two water-based ferrofluids having different nanoparticle concentrations are investigated for Re <100 cases. A parametric study is performed to investigate the effect of the solid volume fraction of nanoparticles, single and multiple dipole configurations and different Reynolds numbers on heat transfer enhancement. Following conclusions can be drawn from the obtained results:

- Laminar forced convective heat transfer by ferrofluids can be significantly augmented by applying external magnetic fields compared to no magnetic field cases. This enhancement is owing to the field-induced localized mixing of ferrofluids due to magnetic body force induced in ferrofluids. This mixing leads to disturbance in the thermal boundary layers thus causing higher bulk fluid temperature which eventually results in the enhancement of the local Nusselt number. Higher solid loading shows better results for all cases because of superior thermo-physical properties of ferrofluids. Placement of multiple dipoles along the channel shows better results for heat transfer enhancement over single dipole case.

- The effect of inertia force over magnetic force can be studied through dimensionless magnetic Froude number. At a critical Reynolds number, the effect of magnetic force becomes insignificant for any magnetic strength, which in turn, does not affect heat transfer significantly. This shows that not all cases of magnetic field application will result in heat transfer enhancement. Hence, it is beneficial to perform a numerical study before real-life experiments to get an estimate of required parameters as well as perceived benefits from the application of external magnetic fields. Numerical simulations are also useful in providing insights on magnetic field gradients and magnetic forces induced by magnetic dipoles in ferrofluids. The spatial range over which magnetic field and forces are effective can be derived from simulation results for the real-life experiments.

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References


Fig. 1: (a) (i-iii) Schematic of problem statement with one, two and three magnetic line dipoles (Strength of 0.5 A-m each) (b) Magnetization ($M-H$) curves of the two ferrofluids at 300 K (plotted using Langevin function shown in equations 7 and 8 for 10 nm particle size)
Fig. 2: (a) Schematic of the problem statement of Ganguly (2004) (b) Comparison of the Nusselt number of present work with their work (c) Validation of current work with Shah and London for no magnetic field cases at different Reynolds number (d) Velocity profile along y-axis at magnet location for different Reynolds number cases for single dipole case for EMG-805 ferrofluid
Fig. 3: (i) Distribution of non-uniform magnetic field \( (H) \) produced by magnetic dipoles (placed at 1 mm separation from the plates) (ii) Contour plots of magnetic body forces \( (f_B) \) induced by magnetic dipoles in EMG-805 ferrofluid (a) Single dipole placed at \( x = 0 \) (b) Two dipoles at \( x = 5 \) mm on both side from center \( (x = 0) \) (c) Three dipoles placement (first at \( x = 0 \) and at 1 mm above the top plate and other two at \( x = 5 \) mm on both side from center)
Fig. 4: (i) Magnetic field ($H$) and (ii) Magnetic body force ($f_k$) on EMG-805 ferrofluid, plotted along the central line of the channel for all three cases of dipole placement.
Fig. 5: Velocity and temperature profiles for (i) EMG-308 and (ii) EMG-805 ferrofluid flow, with and without magnetic field for single dipole case (a) at Re = 25 without magnetic field, all other cases with magnetic field at (b) Re = 25 (c) Re = 50 (d) Re = 75
Fig. 6: Velocity and temperature profile for (i) EMG-308 and (ii) EMG-805 ferrofluid flow, with and without magnetic field for two dipole case (a) at Re = 25 without magnetic field, all other cases with magnetic field at (b) Re = 25 (c) Re = 50 (d) Re = 75
Fig. 7: Velocity and temperature profile for (i) EMG-308 and (ii) EMG-805 ferrofluid flow, with and without magnetic field for three dipole cases (a) at Re = 25 without magnetic field, all other cases with magnetic field at (b) Re = 25 (c) Re = 50 (d) Re = 75
Fig. 8: (i) Local Nusselt number plot (ii) Bulk fluid temperature plot for EMG-805 fluid, with/without magnetic field for single dipole case at Re = 25 (refer fig. 1a(i))
Fig. 9: Local Nusselt number (Nu) plots on top and bottom plates for both the ferrofluids at different Reynolds number; Bottom plate and Top plate Nu for, (a, b) single dipole case, (c, d) two dipoles case, (e, f) three dipoles case.
Fig. 10: Bulk fluid temperature ($T_b$) plots for both the fluids at different Re along the plate length (a) EMG-308 (b) EMG-805; single magnet case, (c) EMG-308 (d) EMG-805; two magnets case, (e) EMG-308 (f) EMG-805; three magnets case
Fig. 11: Average Nusselt number plots for single and multiple dipole cases for both the ferrofluids at different Reynolds numbers, (a) for EMG-308, (b) for EMG-805, ferrofluid
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HIGHLIGHTS:
- Ferrofluids under magnetic field can significantly affect heat transfer.
- Interplay of inertia and magnetic forces affects augmentation potential.
- Higher solid particle loading and multiple dipoles placement produce better results.
- Local Nusselt number observed to reach as high as 200% of no magnetic field case.